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# Two Multiobjective Metaheuristics for Solving the Integrated Problem of Frequencies Calculation and Departures Planning in an Urban Transport System

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## Abstract

*The process of urban public transport planning commonly includes four basic activities, usually executed in sequence: Network design, Timetabling, Vehicle scheduling and Crew scheduling. In this paper we present a multiobjective model that integrates the calculation of minimum frequencies and departures scheduling (minimum frequencies are calculated when solving the Network Design Problem. The calculated frequencies are employed for Timetabling Construction Problem). Two multiobjective metaheuristics for solving randomly generated instances of the problem are presented and their performances are compared. The main scientific contribution of this paper is the development of an integrated mixed integer linear programming model to construct timetable by selecting frequencies in such a way that multiple objectives, like operational cost, synchronization, transfer time and smooth transitions between periods are optimized.*

***Keywords:*** *Integrated, frequency, timetable.*

## 1. Introduction.

In this paper is addressed the public transport planning problem. This is a process which is usually divided into four phases: network design, timetabling construction, vehicle scheduling, and crew scheduling. Usually, these phases are executed sequentially. Here in the paper, we are tackling the bus timetable construction problem of an urban bus transport network. This is usually accomplished in three steps: first for each scenario (covering a concrete planning period) bus frequencies are calculated for each route in the network, then bus departures are settled for each route in the network based on previously calculated frequencies. This is then adjusted for getting acceptable timetables for planners.

The main scientific contribution of this work is the development of an integrated multi-objective mixed integer lineal mathematical model to construct multi-period urban bus timetables, which also allows smooth transitions between

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adjacent planning periods with different demand.

Recently Ibarra-Rojas & Ríos-Solís (2012) have shown formally that the timetabling problem is NP-Hard, so we implemented two multi-objective metaheuristics to explore the effectiveness of the proposed model. We designed an experiment for testing the heuristics with generated random instances.

The timetabling problem has been tackled in the literature from different approaches. Ceder (2007) proposes an exact methods for creating a timetable with maximal synchronization. Also, Eranki (2004), proposes a model to create timetables with maximal synchronization using time windows. She used a heuristic method to solve the problem, but she did not consider multiple criteria. Also, other authors consider maximization of synchronization as a key objective in urban transport planning. Among them, Paunovic (2013) showed a positive correlation between children blood pressure and road traffic noise, transit density and public transport. Burke (2011) also advocates the importance of taking into consideration passenger transfer as a measure of quality for an urban transport system, which indirectly calls for synchronization maximization. Another important measure of quality for urban transport planning is quality of service from the users' perspective (see Ibeas & Cecin, 2011). Ibeas & Cecin, (2011) concluded that the most important variables when defining quality of public transport from the users' perspective are waiting time, journey time and above all, level of occupancy. Recently, this claim has been a subject of research studies by some researchers. For example, Barra et al (2007) presented a model considering different characteristics of the transport system (passenger requirements, budget constraints, level of service). There are other research works on timetabling problem in which the authors use metaheuristics like GRASP. Among these is Mauttone & Urquhart (2009) who developed a metaheuristic based on GRASP for optimizing simultaneously different objectives for passengers and schedulers.

In literature, there are approaches such as Szeto & Wu (2011) that combine two phases of the urban transport process. Szeto & Wu (2011) propose a simultaneously integrated solution for the bus network design and frequency setting problems using a genetic algorithm (GA) that tackles the route network design problem. GA is hybridized with a neighborhood search heuristic which tackles the frequency setting problem. Also in Cipriani et al. (2012), network design and frequency calculation are integrated for optimizing passenger transfer, among other impact measures. There are also approaches for solving two phases sequentially. A good example is Chakroborty (2003) who combines the transit routing and scheduling phases using a genetic algorithm. In his approach, he tries to minimize the transfer time and the waiting time. Another research that combines several phases is the one proposed by Zhao & Zeng (2008). Zhao & Zeng (2008) present a metaheuristic method for optimizing transit networks, including route network design and vehicle headway and timetable. The goal is to identify a transit network that minimizes a passenger cost function. Their metaheuristic combines simulated annealing, tabu and greedy search methods.

In some published research works, multiple criteria are considered. In some others, different phases of the transport system are combined sequentially or integrated. Some others, smooth transitions between periods are considered. An example of these is Ceder (2007) who proposes two techniques to handle the smooth transitions between periods with different demand. We have not come across any article that integrates minimum frequency problem and the timetabling problem and considers multiple objectives and the smooth transition simultaneously.

The rest of our paper is organized as follows: in section 2, we will describe the problem, present the mathematical model and give a brief description of it. In section 3, we will describe the decision support methodology we are implementing. Some results will be presented in section 4. In section 5, we will present some discussions about ranking portfolios. Finally, in section 6, we will present our conclusions and suggestions for future research.

## 2. The Development and Interpretations of the Model

According to Ceder (2007), the transport planning process is divided into four phases: network design, timetabling, vehicle scheduling and crew scheduling. The timetabling phase has two activities: frequency determination and timetable assignment. These activities are executed sequentially.

The problem addressed in this paper relates to the development of a mathematical model for determining, in an integrated way, the frequencies and the timetables for the operation of the urban transport. The model is a multi-period model with changing demands.

Additionally, there are multiple objectives that have to be considered in the model. The objectives are derived from the requirements of the social actors involved in the process: like synchronization (between bus routes in a specific node), operational transport cost, transfer time and smooth transitions between adjacent periods (a transition from a period with high demand to a period with low demand or the other way).

### *2.1. The Development of the Model*

Assumptions:

The assumptions on which the model is based are:

- Demand does not change significantly in each period and it is known in advanced.
- Average travel time from each route in each period is known.
- Periods lengths must be enough to allow the schedule of the needed departures.
- The planning requirements must ensure the satisfaction of the demand during the planning period established.

- We consider only departures from the same period of the synchronization we wish to activate.

Sets:

The following set notations are used in the model:

$M$: Set of routes.
$K$: Set of nodes.
$V$: Set of periods.
$B_{ij}^{v}$ Set of pairs of nodes where potentially synchronize the routes $i$ and $j$.
$J(i)$: Set of routes which have common nodes with the route $i$

Variables:

The following are the model's decision variables:

$X_{ip}^{v} = 1$: There is a trip in the route $i$ with departure time in the interval $\left(p \cdot H_{min_i}^{v}, p + 1 \cdot H_{min_i}^{v} + g\right)$in the period $v$ y $0$ otherwise.
$\alpha_{ip}^{v} \in \left(p \cdot H_{min_i}^{v}, p + 1 \cdot H_{min_i}^{v} + g\right)$ iff $X_{ip}^{v} = 1$, $\alpha_{ip}^{v} = 0$ iff $X_{ip}^{v} = 0$
$Y_{ijkupq}^{v} = 1$ If the bus of the route $i$ with departure time in the interval $\left(p \cdot H_{min_i}^{v}, p + 1 \cdot H_{min_i}^{v} + g\right)$ and the bus of the route $j$ with departure time in the interval $\left(q \cdot H_{min_i}^{v}, q + 1 \cdot H_{min_i}^{v} + g\right)$ in the period $v$, arrive to the segment $k - u$ (fixed synchronization node) within the window time and $0$ otherwise.
$\mu_{ip}^{v}$: Represents the absolute difference in relation to the closer departure time of the even average loads method if there is a trip in the route $i$ in the interval $\left(p \cdot H_{min_i}^{v}, p + 1 \cdot H_{min_i}^{v} + g\right)$ in the period $v$.
$Z_{ijku}^{v}$: The difference between the arrival time of the routes $i$ and $j$ in the segment $k - u$ in the period $v$.

Parameters:

The following are the parameters of the model:

$G^{v}$: Number of trips in the period, if we use a frequency equal to $H_{max_i}^{v}$.
$P_{max_i}^{v}$:Maximum load of passengers in the route $i$ in the period $v$.
$P_{maxd_i}^{v}$:Maximum load of passengers on bord in the day in the route $i$.
$d_i^{v}$:Desired occupancy of the bus in the route $i$ in the period $v$.
$Pas_i^{v}$: Total passengers/km, in the route $i$ in the period $v$.
$L_i$: Length of the route $i$.
$cap_i^{v}$: Bus capacity of the route $i$ in the period $v$.
$l_k$: Length of the segment $k$.
$\beta_i^{v}$: Percentage allowed of the route $i$ of exceed the load in the period $v$.
$H_{min_i}^{v}$: Minimum headway of the route $i$ in the period $v$.

$H^v_{max_i}$: Maximum headway of the route *i* in the period *v*.
$T^v$: Planning period $\left[T^v_{ini}, T^v_{fin}\right]$.
$T^v_{ini}$: Beginning time of the planning period *v*.
$T^v_{fin}$: Ending time of the planning period *v*.
$\gamma^v_i$: Desired time before the end of the period $T^v$ for the last departure of the route *i* in the period *v*.
$W^v_{max_i}$:Maximum window time for the route *i* in the period *v*.
$W^v_{min_i}$:Minimum window time for the route *i* in the period *v*.
$t^v_{ik}$Travel time from the origin point of the route *i* to the segment *k* in the period *v*.
$\delta^v_{ijku}$:Minimum time the passenger needs to change from segment *k* of the route *i* to the segment *u* of the route *j* in the period *v*.
$\pi^v_{ijku}$: Number of passengers changing from segment *k* of route *i* to the segment *k* of the route *j*.
$\overline{P^v_{max_i}}$Maximum load average of passengers on bus of route *i* in the period *v*.
$MC^v$Method applied to determine the frequency in the period *v*.
$fmr^v_i$:Minimum frequency required to satisfy the demand of the route *i* in the period *v*.

$$fmr^v_i = \frac{\overline{P^v_{max_i}}}{d^v_i}$$

$C^v_{ip}$:Timetable calculated with the even average load method. 1 if there is a departure in the interval *p* for the route *i* in the period *v*.
$FixedCost^v_i$: Fixed cost for the route *i* in the period *v*.
$VariableCost^v_i$:Variable cost for the route *i* in the period *v*.
$P^v_k$: Average of passengers on board in the segment *k* in the period *v*.
$s^v_{jk}$:Holding time of the route *i* in the interval *p* during the period *v*.

The model:

Using all the set symbols, decision variables, and input parameters defined above, we develop the model's objective functions and constraints and present it (the model) as follows.

$$min \sum_{i\in M} \sum_{v\in V} \left(FixedCost^v_i + VariableCost^v_i \cdot L_i \cdot \sum_{p\in N^v} X^v_{ip}\right) \quad (1)$$

$$max \sum_{i\in M} \sum_{j\in J(i)} \sum_{(k,u)\in B^v_{ij}} \sum_{v\in V} \sum_{p\in N^v} \sum_{q\in N^v} Y^v_{ijkupq} \quad (2)$$

$$min \sum_{i\in M} \sum_{j\in J(i)} \sum_{v\in V} \sum_{(k,u)\in B^v_{ij}} \pi^v_{ijku} \cdot Z^v_{ijku} \quad (3)$$

$$min \sum_{v\in V} \sum_{i\in M} \sum_{p\in N^v} \mu^v_{ip} \quad (4)$$

s.t.

$$MC^v = 1 \Rightarrow \sum_{p \in N^v} X_{ip}^v \geqslant \frac{P_{maxd_i}}{d_i^v}; v \in V; i \in M \tag{5}$$

$$MC^v = 2 \Rightarrow \sum_{p \in N^v} X_{ip}^v \geqslant \frac{P_{max_i}}{d_i^v}; v \in V; i \in M \tag{6}$$

$$MC^v = 3 \Rightarrow \sum_{p \in N^v} X_{ip}^v \geqslant \frac{Pas_i^v}{d_i^v \cdot L_i} \wedge \sum_{p \in N^v} X_{ip}^v \geqslant \frac{P_{max_i}^v}{cap_i^v}; v \in V, i \in M \tag{7}$$

$$MC^v = 4 \Rightarrow \sum_{p \in N^v} X_{ip}^v \geqslant \frac{Pas_i^v}{d_i^v \cdot L_i} \wedge \sum_{p \in N^v} X_{ip}^v \geqslant \frac{P_{max_i}^v}{cap_i^v} \wedge$$
$$\sum_{l \in I^v} l_k \leq \beta_i^v \cdot L_i; v \in V, i \in M, I^v = \left\{k \vee \frac{P_k^v}{fmr_i^v} \geqslant d_i^v\right\} \tag{8}$$

$$X_{ip}^v = 0 \Leftrightarrow \alpha_{ip}^v = 0; \forall v \in V, \forall i \in M, \forall p \in N^v \tag{9}$$

$$\sum_{p \in N^v} X_{ip}^v \geqslant max\{G^v, fmr_i^v\}; v \in V, i \in M \tag{10}$$

$$\alpha_{ip}^v \leq \alpha_{il}^v \wedge \alpha_{il}^v > 0 \Rightarrow \alpha_{ip}^v \leq H_{max_i}^v; \forall v \in V, \forall i \in M, \forall p, l \in N^v \tag{11}$$

$$\alpha_{ih}^v > 0 \wedge \alpha_{ip}^v > 0 \wedge \alpha_{ip}^v = min(\alpha_{il}^v) \Rightarrow \alpha_{ih}^v + H_{min_i}^v \leq \alpha_{ip}^v \leq \alpha_{ih}^v + H_{max_i}^v;$$
$$\forall v \in V, \forall i \in M, \forall p, h \in N^v, l > h \tag{12}$$

$$\alpha_{ip}^v > 0 \wedge \alpha_{ip}^v = max(\alpha_{il}^v) \Rightarrow T_{fin}^v - \gamma_i^v \leq \alpha_{ip}^v; \forall v \in V, \forall i \in M, \forall p, l \in N^v \tag{13}$$

$$Y_{ijkupq}^v = 1 \Rightarrow \alpha_{ip}^v > 0 \wedge \alpha_{jq}^v > 0 \wedge W_{min_i}^v - t_{ju}^v - s_{jk}^v + \alpha_{ip}^v + t_{ik}^v + \delta_{ijk}^v \leq \alpha_{jq}^v$$
$$\wedge \alpha_{jq}^v \leq W_{max_i}^v - t_{ju}^v - s_{jk}^v + \alpha_{ip}^v + t_{ik}^v + \delta_{ijku}^v \wedge \alpha_{jq}^v + t_{ju}^v + s_{jk}^v \geq \alpha_{ip}^v + t_{ik}^v + \delta_{ijk}^v$$
$$v \in V, i \in M, (k,u) \in B_{ij}^v j \in J(i), p \in N^v, q \in N^v \tag{14}$$

$$Y_{ijkupq}^v = 1 \Rightarrow Z_{ijku}^v = max\left(\left(\alpha_{jq}^v + t_{ju}^v + s_{ju}^v\right) - \left(\alpha_{ip}^v + t_{ik}^v + \delta_{ijku}^v\right)\right) \tag{15}$$
$$v \in V, i \in M, (k,u) \in B_{ij}^v, j \in J(i), p, q \in N^v$$

$$Y_{ijkupq}^v = 0 \Rightarrow Z_{ijku}^v = 0; v \in V, i \in M, (k,u) \in B_{ij}^v, j \in J(i), p, q \in N^v \tag{16}$$

$$\alpha_{ip}^v > 0 \Rightarrow \mu_{ip}^v = min\left|\left(C_{is}^v - \alpha_{ip}^v\right)\right|; v \in V, i \in M, (k,u) \in B_{ij}^v; p, s \in N^v \tag{17}$$

$$\alpha_{ip}^v = 0 \Rightarrow \mu_{ip}^v = 0; \forall v \in V, \forall i \in M, \forall p \in N^v \tag{18}$$

$$X_{ip}^v \in 0{,}1; \alpha_{ip}^v \in T_{ini}^v, T_{fin}^v; Y_{ijkupq}^v \in 0{,}1; Z_{ijku}^v \in \mathbb{R}; \mu_{ip}^v \in \mathbb{R}$$
$$\forall i, j \in M; \forall k, u \in K; \forall v \in V, \forall p, q \in T^v$$

### *2.2. Model Interpretations*

The model consists of 4 objective functions, the first objective function, (1), minimizes the total cost. There are fixed and a variable costs associated with the long route and the departures made on the route in any period. The second function, (2), maximizes the number of synchronizations between two bus routes in a period. The third function, (3), minimizes the transfer times, and the fourth function, (4), minimizes a penalty for not meeting the departure time obtained with an average loads method (Brans & Mareschal, 2005), which guarantees a good transition between periods with different demand.

These objective functions are subjected to frequency constraints (5) to (8), which were proposed by Ceder (2007). Constraint (9) says that if there is no travel in period *v* on the route *i* in the segment *k*, then we do not assign a departure time. Constraint (10) ensures that the quantity of departures must be the maximum of the number of departures determined by the maximum headway and the minimum frequency, which satisfies the maximum load point. This guarantees that demand is met.

Constraint (11) specifies that the departure time of the first departure must be less or equal to the maximum headway. Constraint (12) is for the consecutive departures. It specifies that the departure time must be between a minimum and a maximum headway. For the last departure, (13) ensures that the departure time must be between the end of the period and a desired time. Constraint (14) represents synchronization. This means when two buses of different routes arrive to a synchronization node between a time window, and taking into account the transfer times, the permanence time in a node, and the travel time, then there is a synchronization.

Constraints (15) and (16) account for the time that passengers wait to do the transfer. Constraints (17) and (18) are related to objective function (4). They represent the difference between the departure assigned by our model and the closer departure, just as in the method of average loads proposed by Ceder (2007).

## 3. Decision support methodology.

The decision making process proposed by Simon (1997) has four phases: intelligence phase, design phase, choice phase, and implementation phase. In the intelligence phase, the reality is examined and the problem is identified and defined. During the design phase, we set up a representative model. The model is validated and the criteria are selected. The choice phase includes a solution to the model. The final phase is the implementation phase, in which the solution to the original problem is implemented.

We applied the first three phases in the decision making process. It should be noted that in our case, the implementation phase is not addressed. In Figure 1, the intelligence phase is covered with the mathematical model presented in the previous section, the design phase covers the optimization - in this case, metaheuristics optimization (MOTS Hansen (1997) and SSPMO Molina et al. (2007)) is applied.

Finally, in the selection phase, we employ Promethee Brans & Mareschal (2005) because the generated ranking of alternatives offered allows the schedulers to choose the most attractive alternatives with regard to his own preferences. Also, it allows the application of any ranking method as an interactive method.

| Phases | | Actions |
|---|---|---|
| Intelligence | → | Mathematical model |
| Design | → | Multiobjective optimization |
| Selection | → | Exploring efficient frontier |

**Figure 1.** Phases of decision making process according to Simon (1997)

MOTS Hansen (1997) is an adaptation of the well know tabu search. It is used heuristically to generate non-dominated alternatives to multiobjective combinatorial optimization problems. MOTS works with a set of current solutions which, through manipulation of weights, are optimized towards the non-dominated frontier while at the same time seek to disperse over the frontier.

The basic MOTS procedure starts by setting a random feasible starting solution and then determining a weight vector for the point. Each element in the weight vector is set according to the proximity of the other points for that objective. The closeness is measured by a distance function based on some metric in the objective function space and using the range equalization weights. The standard tabu search procedure is used to replace a current solution with the best feasible neighbor solution that is determined by the scalar product between the weight vector and the vector objective function. The new point is inserted into the ND-set if it is non-dominated. Then we replace one randomly selected solution by another randomly selected solution whenever a drif-criterion is reached and we continue with the next iteration until a stop-criterion is met.

SSPMO Molina et al. (2007) consists of a scatter/tabu search hybrid that includes two different phases: 1) generation of initial set of efficient points through various searches and 2) combinations of solutions and updating of efficient frontier ($\hat{E}$) via scatter search.

The procedure starts by linking *p+1* tabu searches. The first tabu search starts from an arbitrary point and attempts to find the optimal solution to the problem with a single objective function $f_1(x)$. Let $x_1$ be the last point visited at the end of this search. Then, a tabu search is applied again to find the best solution to the problem with the single objective $f_2(x)$ using $x_1$ as the initial solution. This process is repeated until all the single-objective function problems associated with the p objectives have been solved. At this point, we again solve the problem with the first objective function $f_1(x)$ starting from $x_p$, to finish a cycle around the efficient set. The aim in this step is to minimize a function that measures the distance to the ideal point.

In the second phase, the main search mechanism is the combination of solutions that are currently considered efficient and therefore belong to $\hat{E}$. The solutions to be combined are selected from the reference set. Every solution that is added to RefSet is also added to Tabu-RefSet. All pairs of solutions in RefSet are combined and each combination yields four new trial solutions. Then the same tabu search used in the initial phase is applied to improve new trial solutions, guided by a compromise function. Solutions generated during this improvement phase are tested for possible inclusion in $\hat{E}$. With this, we update the RefSet in preparation for the next scatter search iteration

After we have presented the procedure of both metaheuristics, we can see that the structure of SSPMO helps us to find better solutions because, first, we construct the space where all possible feasible solutions can be when we solve individually each objective and we use the best solution to star the next search and then we improve the found solutions. Using MOTS, we generate a set of feasible solutions by optimizing all objective at the same time, and we do not have an idea of where best solutions could be.

## 4. Results.

Random instances were created and classified according to the number of periods, bus stops and routes into small, medium and large. In relation to synchronization, the instances were classified by density according to the percentage of the combinations of bus stops in each route. (See Table 1). The instances generator was developed in OPL (Optimization Programming Language).

**Table 1.** Characteristics of instances.

| | Small | Medium | Large |
|---|---|---|---|
| Routes | 2-4 | 5-8 | 8 |
| Periods | 3-5 | 8-10 | 8-10 |
| Nodes | 10-18 | 19-23 | 35-50 |
| Density | 1%-2% | 2%-4% | 4%-7% |

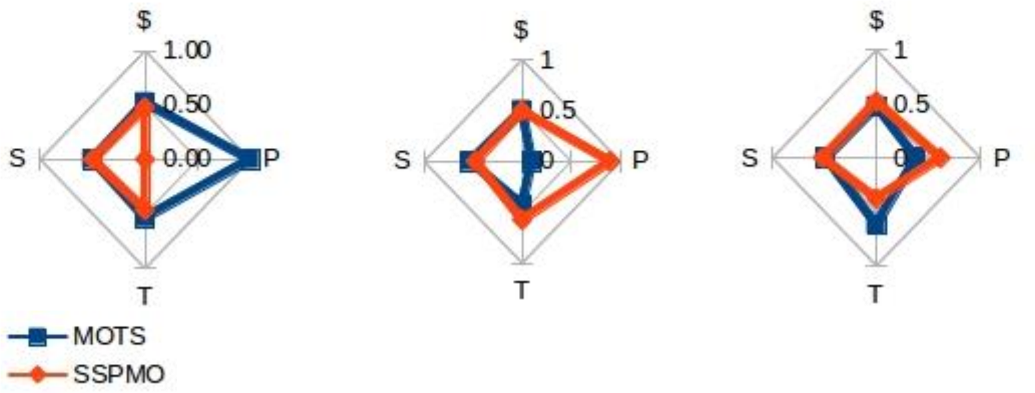
Out of the 25 instances tested, we have selected three of them to show their results and compare both algorithms because in the other instances the behavior is similar. Table 2 shows the classification random instances.

**Table 2.** Results of the classifications of random instances

| | Small | Medium | Large |
|---|---|---|---|
| Routes | 3 | 4 | 6 |
| Periods | 5 | 9 | 10 |
| Nodes | 37 | 88 | 153 |
| Headways | 9-19, 11-17, 10-18,8-20 | 6-10,4-12 | 6-10,4-12 |
| Synchronization nodes | 45 | 271 | 1201 |
| Waiting Window | 10-29, 5-16, 6-33, 10-14,3-29 | 4-34,4-18, 10-26,1-16, 10-29,2-22, 7-28,3-15, 5-34 | 7-28, 7-18, 5-28, 4-19, 9-27, 1-21, 10-30, 2-19, 13-29, 8-15 |

For the medium instance category, we got 19 efficient solutions with MOTS and 99 efficient solutions with SSPMO. We find that most of the objectives in both methods have a distance very similar to the ideal point (center of graph). However, the distance of the penalty obtained with SSPMO in this instance is bigger.

In the large instance category, we got 21 efficient solutions with MOTS and 99 efficient solutions with SSPMO. We find that both methods have very similar distances in cost and synchronizations but the distance for penalty and transfer time make both solutions attractive for the decision maker according to his preferences.




Small Instance Medium Instance Large Instance

**Figure 2.** Distance to the ideal point

In Table 3, we present the summary of the results we obtained in each instance category with each metaheuristics. The execution time for each metaheuristic is also presented.

**Table 3.** Results obtained in each instance category with each metaheuristic

| O.F. | Small | | Medium | | Large | |
|---|---|---|---|---|---|---|
| | MOTS | SSPMO | MOTS | SSPMO | MOTS | SSPMO |
| Cost | .51 | .49 | .50 | .50 | .50 | .50 |
| Synchronization | .47 | .53 | .46 | .54 | .50 | .50 |
| Transfer Time | .49 | .51 | .47 | .53 | .50 | .43 |
| Penalty | 1 | 0 | .10 | .90 | .68 | .32 |
| Ex. Time (sec.) | 30 | 84420 | 1320 | 2704620 | 2820 | 124140 |

## 5. The Comparison and Ranking of the Solutions

To compare the solutions obtained with both metaheuristics, a metric method, a ranking method or an interactive method like Korhonen & Halme (1996) can be used. In our case, we used a ranking method. We decided to implement a Promethee I, for simplicity, but we would like to note that any other method can be selected.

To establish the preference values, we simulated a decision maker. We consider 5% as the indifference threshold and 20% as the preference threshold for all objectives. The determination of these values are based on the range of variation of the values of the objectives of the alternatives. With regard to the weights, the cost is considered as the main objective and, therefore, a weight of 50% is assigned to it. The synchronizations and the transition between periods are equally important. Hence, a weight of 20% is assigned to each of them. The least important is the transferring time which has a weight or 10% attached to it.

In Figure 3, we made comparisons between MOTS and SSPMO based on an outranking relation. These results have been obtained by applying Promethee I to the set of alternatives, conformed for the efficient solutions given for both methods in a particular instance. In each column we present blocks of efficient solutions obtained with both methods, the superior block outranks the solutions of the inferior block.

The example for the small instance category, the first block, has 138 efficient solutions of SSPMO, these solutions outrank the three solutions of the second block, but these three solutions outrank the solutions of the third block, and so on. From these results, one can conclude that SSPMO gives solutions of better quality (with respect to the proximity of Pareto front) than MOTS.

These comparisons provide the criteria for discarding non-efficient solutions. If an efficient solution generated by one of both methods is outranked by at least one solution of the other method, then it will be discarded.

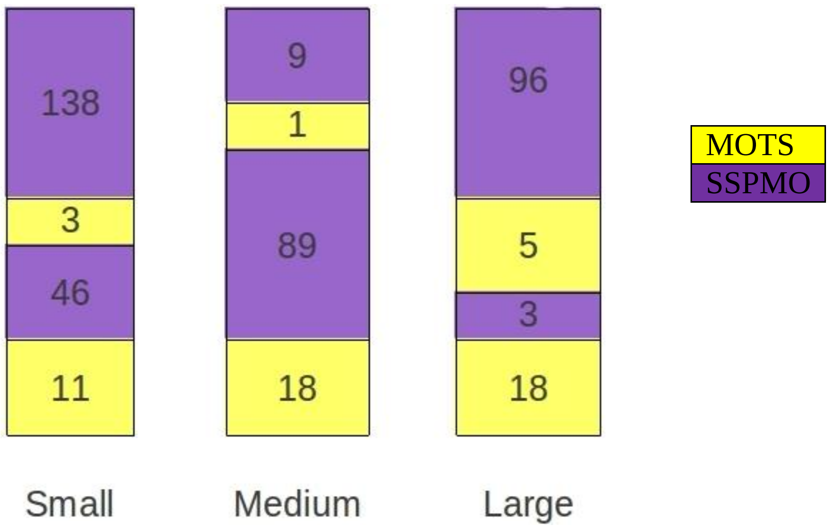



**Figure 3.** Outranked solutions

These results are by no means conclusive, and could be very different for different decision makers. They are included here only for illustrative purpose.

## 6. Summaries & Conclusions.

### *6.1. Summaries*

In this work it have been defined for the first time a mathematical model for urban bus planning process that includes characteristics as: integrated frequency and timetabling, considering multiple periods with smooth transitions between periods with different demands and multiple objectives representing interests from all social actors involved. It also have been defined the limits and the scope of this model by the establishment of a set of assumptions that determined the validity in the application of the model in real cases.

We developed a decision support methodology to assist the decision maker in the first three phases of the decision making process, namely: helping him to structure the problem, to establish his preferences and to choose rationally those solutions with an acceptable trade off among different objectives.

The solutions obtained with SSPMO have better quality that was achieved with an execution time significantly higher than the time needed by MOTS for solving the same instances.

### 6.2. Conclusions

The frequency and timetable integrated problem is a NP-Hard problem that has not been studied enough. They need deeper and more extensive studies due to their importance in the planning of urban transport systems.

In Mexico, it is very important to develop a tool which solves this problem because the decision makers develop schedules based on their experience. If there are tools or techniques that can be applied, transport agencies will be able to improve their performance, minimize costs and give better quality services to their passengers or customers. Presently, we do not know of any attempt that has been made to solve the integrated frequency and timetable problem.

Here in our research, we have implemented two metaheuristics for solving the frequency and timetable integrated problem. Among them, SSPMO is the one that give us better results.

Although we present a mathematical model in this research, we have not developed or implemented an exact method for its solution. We hope to do that in future. In this research, we have assumed that demand is deterministic. But, in real life, this is not always the case. In future, we would like to consider and incorporate situations in which there are demand and travel time uncertainties.

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