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Weight Stability Intervals in Multicriteria Decision Aid Under Semiorder Preference Structures

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Abstract

One essential problem in Multicriteria Decision Aid is to assess the relative importance of different criteria. The use of weights gives the decision-maker the possibility to better modelize the real aspects of a decision problem and to express more freedom the preference structure he has in his mind. This task is not easy because a subjective component is always present and there exist great number of methods which try to approximate this problem. PROMETHEE Methods consider as outranking non-compensatory methods; give the possibility to calculate weight stability intervals. So it is very important to do sensibility analysis taking into account that changes in weights would be reflected in PROMETHEE decision axis and they could affect previous conclusions. The idea of weight stability intervals (WSI) was introduced by Mareschal (1988) in PROMETHEE Methods. It is well known that these methods work under a preorder preference structure, so we propose to calculate the WSI under a semiorder structure with the aim to study the stability and the robustness of the model from a more solid point of view. In this paper we propose to analyze in a first-order additive method, to say, with only one valued real function, specifically in PROMETHEE II, the sensibility of a Semiorder Preference Structure under changes in the weight vector. The task consists of defining New Weight Stability Intervals (NWSI) adapted to a Semiorder Preference Structure in PROMETHEE Methods.

Keywords*: *weight stability interval, semiorder preference structure, PROMETHEE Methods, thresholds.

1. Introduction

In multiple criteria decision aid the assessment of the relative importance of the different criteria plays a crucial role. A number of methods which are mainly focused on the definition, determination and influence of the criteria weights have been proposed in literature. Criteria weight is a kind of quantity that tries to express the decision maker's subjective preference but the definition itself is not always very precise.

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Due to the fact that decision makers are not very clear to assign a weight each criterion in the beginning, the weight of the criteria will vary continuously through the process of the ranking of alternatives and the ranking results may change with the alteration of the weight.

It is true that the use of criteria weight gives the opportunity to a decision maker to modelize or express his feeling/judgment about a decision problem but it is necessary to be very careful when determining criteria weights. To help decision makers understand how the change of weight influence the ranking results, the concept of weight stability intervals for the weights of different criteria was introduced by Mareschal (1988b) and the PROMETHEE Methods where selected to operationalize the new approach.

The extension of the PROMETHEE Methods that we have proposed introduces a preference structure that is even more complex than the traditional one's which notably enrich the modelization phase: a Semiorder Preference Structure. In this way, the Weight Stability Intervals requires a new definition in correspondence with the preference structure.

It will be very interesting to study the extent to which the consideration of Semiorder Preference Structures within PROMETHEE Methods improves the decision making process in its entirety.

Having exhaustively analyzed the referred preference structure, it can be said that it gives more flexibility, amplitude and certainty to the preference formulations, as they tend to abandon The Complete Transitive Comparability Axiom of the Preferences to replace it by the Partial Comparability Axiom of the Preferences. Going form an axiom to the other allows us to introduce, in the analysis, the Incomparability that are basically present when: (1) the decision-maker is not able to discriminate between two alternatives since the information that he has, is too subjective or too incomplete to produce a judgment of Indifference or Strict Preference; (2) the decision-maker is in a position that not allow him to determine the preferences since the last responsible for the decision may be inaccessible, being either a remote entity or a loose entity with ill-defined and/or contradictory preferences; and (3) the decision-maker does not want to discriminate and he prefers to remain removed from the decision process and wait until a later stage when he has more reliable and sure information about the preferences.

The New Weight Stability Intervals (NWSI) is presented in section 2. We develop the new intervals in PROMETHEE II which is considered as an additive method of first order but under a semiorder preference structure. Different types of sensibility are defined and studied.

In order to point out the contributions of PROMETHEE methods with NWSI under a Semiorder Preference Structure different numerical applications are presented in Section 3.

2. New Weight Stability Intervals (NWSI) and PROMETHEE Methods

The use of sensibility analysis has been introduced in PROMETHEE Methods in order to help facilitate the interpretation of the results. In this way it is possible to study the consequences of the modifications of initially specified weights on the results. These sensitivity analyses require the determination of weight stability intervals, polygons and areas (Mareschal, 1989). On one hand, they provide sufficient information on the stability of the ranking and, on the other hand, they do no give insight in the way the ranking changes if the stability limits are exceeded. Therefore, it is interesting to know the minimum modification of the weights required to modify the ranking in a certain way.

Weight Stability Intervals in the original formulation are in correspondence with the preference structure of the PROMETHEE Methods (Brans & Mareschal, 1994), that is, a preorder preference structure.

In this paper we propose to analyze in a first-order additive method with only one valued real function, specifically in PROMETHEE II, the sensibility of a Semiorder Preference Structure under changes in the weight vector (Fernández, 1993). The task consists of defining New Weight Stability Intervals (NWSI) adapted to a Semiorder Preference Structure in PROMETHEE Methods (Fernández, 1995).

Three types of stability will be distinguished depending on decision maker requirements: full stability; partial stability; set of good alternatives stability.

The study of the weights stability with a more complex structure enriches the analysis that offers not only PROMETHEE to GAIA Visual Modelization Technique as well. A lot of time would be saved and the decision-maker would have a meaningful participation if he selects or specifies the type of stability wished.

2.1. PROMETHEE II: Additive Method of first order under Semiorder Preference Structure

Let us consider a finite set A of feasible alternatives, and k real-valued criteria $\{g_1, g_2, \dots, g_k\}$ to be maximized (it also can be minimized). The weight of each criterion g_j is a positive real number denoted as w_j, which represents the relative importance of g_j for the decision maker.

To help the decision maker in selecting the "best alternatives" or ranking all these alternatives from the best to the worst, a preference relation or a preference structure will be built on A [*(P: Preference, I: Indifference, R: Incomparability)*]. In this paper we will consider a Total Semiorder or a Partial Semiorder (Roubens & Vincke, 1985).

2.1.1. Preference Structure: Semiorder

An additive method is of order r if and only if there exist r real valued functions of the form:

$$V_p(a) = \sum_{j=1}^{k} w_j\, V_{pj}\big(g_j(a), g_j(A)\big) \quad a \in A; \quad p = 1, \ldots r, \quad \text{such that } \forall\, a, b \in A:$$

$$\begin{cases} a\,P\,b \;\Leftrightarrow\; V_p(a) > V_p(b) + q_j \quad p = 1, \ldots r \quad 0 \le q_j\,,\; q_j \text{ is the indifference threshold} \\ a\,I\,b \;\Leftrightarrow\; -q_j \le V_p(a) - V_p(b) \le q_j \\ \quad\Leftrightarrow\; \begin{cases} V_p(a) \le V_p(b) + q_j & \text{or} \\ V_p(b) \le V_p(a) + q_j & p = 1, \ldots r \end{cases} \\ a\;R\;b \quad \text{otherwise} \end{cases} \tag{1}$$

The notation $\left[V_{pj}\big(g_j(a), g_j(A)\big)\right]$ emphasizes the dependence of V_{pj} not only on $g_j(a)$ but also possibly on the evaluations of all the other alternatives through criterion g_j.

We will study the case where *(P, I, R)* is a total semiorder therefore $r = 1$.

If we consider that PROMETHEE I outranking method constructs a partial semiorder instead of a partial preorder, two functions must be noted (Bans; Vincke, 1985)

The leaving flow: $\phi^+(a) = \sum_{j=1}^{k} w_j \phi_j^+(a) \qquad \phi_j^+(a) = \sum_{b\,\in A} P_j(a,b)$ (2)

The entering flow: $\phi^-(a) = \sum_{j=1}^{k} w_j \phi_j^-(a) \qquad \phi_j^-(a) = \sum_{b\,\in A} P_j(b,a)$ (3)

Where $P_j(a,b)$ are the unicriterion preference functions and $\sum_{j=1}^{k} w_j = 1$ (normalized weights).

The PROMETHEE II total semiorder is obtained by considering the net flow

$$\phi(a) = \left[\phi^+(a) - \left(\phi^-(a) + \boldsymbol{Q}\right)\right] \quad \boldsymbol{Q} = \frac{1}{n}\left[\sum_{1=1}^{n} \frac{q_i}{Máx.g_i}\right] \tag{4}$$

$$4\phi(a) = \left[\sum_{j=1}^{k}\big(w_j\, \phi_j(a)\big) - \frac{1}{n}\left[\sum_{1=1}^{n} \frac{q_i}{Máx.g_i}\right]\right] \tag{5}$$

W here $\boldsymbol{Q} = \left(\frac{\mathbf{1}}{\boldsymbol{n}}\right)\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}}\left(\frac{\boldsymbol{q_i}}{\mathbf{max}\,\boldsymbol{g_i}}\right)$ is the Threshold of Outranking Indifference which is defined as the arithmetic mean of the values that express the relative importance of each criterion g_i.

The $\boldsymbol{Q}$ parameter whose determination requires the interactivity between the analyst and the decision maker, indicates the bigger value under which it exists an indifference feeling among the outranking character of the alternatives in such a way that, the outranking power of alternative *a* is indifferent to the outranking power of alternative b, whenever it does not outrank the Indifference threshold $\boldsymbol{Q}$ (Fernández, 1998).

Besides, if we let: $V_{ik}(a) = \phi_j(a)$ we can conclude that PROMETHEE II is an additive method of order 1. That is, the net flows of $a \in A$ evaluated under g_j criteria represent the value function that allows characterising PROMETHEE II as additive of order 1.

To work with net flows implicitly reflects the New Preference Structure (Semiorder) due to its determination to take part of the indifference thresholds (Wolters & Mareschal, 1995; Fernández, 2002)

Having introduced the preference structure *(P, I, R)* as a Total Semiorder $(R = \emptyset)$, we will study the sensibility of this structure induced by variations of the weights. That is, we will study the stability of the results taking into account the three types of stability mentioned above (Fernández et al., 1997; Roy & Vincke, 1987).

2.2 New weight Stability Intervals (NWSI) for a single criteria

2.2.1. The Model

In our model we will define the modification that will be introduced in the NWSI construction. We assume that the MCDA method is additive of order 1 (Mareschal, 2013).

The weights of the multicriteria must be strictly positive. In any case in which a weight has value zero, the corresponding criterion is irrelevant and it could be deleted. The weights are normalized to 1: $\sum_{j=1}^{k} w_j = 1$.

The objective is to investigate what the Semiorder Preference Structures becomes when all the weights of the criteria are kept constant except for one criterion, say g_j.

The different weights are not fixed; they must be increased or decreased from their initial values. In that way, the NWSI for criteria g_j represents the bounds within which stability is achieved.

It must be recognized that the internal structure of the NWSI differs from the structure of the traditional WSI which has a preorder preference structure associated.

When all the weights of the criteria are kept constant except for one criterion, say g_i, the NWSI of this criterion is denoted by: $w_j' = (1+\beta)w_i$, $\beta \geq -1$, where w_j' are the modified weights. In order to keep the modified set of weights normalized, it is necessary to adjust all the other weights in the following way, ensuring that only the importance of g_i, relative to the other criteria is modified: $w_j' = \alpha w_j,\ j \neq i$.

The relation between α and β parameters as well as constrains on their values to ensure non-negativity of the modified weights are expressed in the following way:

$\alpha = \frac{1-(1+\beta)w_i}{1-w_i}$ (6) $\alpha = \frac{1-w_i'}{1-w_i}$ (7) $-1 \leq \beta \leq \frac{1-w_i}{1-w_i}$ (8)
$0 \leq \alpha \leq \frac{1}{1-w_i}$ (9)

The value function for the modified weights w_j' is denoted by V ‘ and it is given by:

$$V_p'(a) = \alpha V_p(a) + (1-\alpha)V_{pi}\left(g_i(a), g_i(A)\right)$$

The expression could be simplified to obtain $V_{pi}\left(g_i(a)\right) = V_{pi}(a)$, so it can be written as:

$$V_p'(a) = \alpha V_p(a) + (1-\alpha)V_{pi}(a)$$

2.2.2. Additive method of order 1: Total Semiorder

The *(P, I, R)* structure is a Complete or Total Semiorder *(P, I)* where, p = 1, and r =1:

$$a\,P\,b \iff V(a) > V(b) + \boldsymbol{Q}$$
$$a\,I\,b \iff \begin{cases} V(a) \leq V(b) + \boldsymbol{Q} \text{ or} \\ V(b) \leq V(a) + \boldsymbol{Q} \end{cases} \quad (10)$$

The Complete Semiorder Structure *(P', I')* associated will have the modified weights w'_j. If we consider two alternatives $a, b \in A$, the following changes can appear:

a P b and *b P' a*: the preference is inverted
a P b and *a I' b*: the preference becomes an indifference
a I b and *a P' b*: the indifference becomes a preference

Which of the three previous changes is the most serious?

Of course, the first change could have bad consequences in the final results. The other two cases are less important; they imply the indifference and it is not relevant enough in the additive MCDA method we are considering here.

Regardless, in our approach we can consider either only the inversions of Preference or all possible changes in both Preference and Indifference.

It must be noted out that the changes in the Preferences (the first type of change) occurs less often in the Semiorder Preference Structure than in the Preorder Preference Structure, so that we could denote an important contribution to the robustness of the model proposed (Rosenhead, 2001a, 2001b).

The condition for inversion of preference between a and b can be formulated as following:

$$\{V(a) - V(b) - \boldsymbol{Q}\}\,\{V'(a) - V'(b) - \boldsymbol{Q}\} < 0 \qquad (11)$$

Even though the other two situations are not taken into account by (11), it is possible to give a stability condition depending on whether a and b are indifferent or not.
Let us denote:

$$\begin{aligned} \Delta(a,b) &= \{V(a) - [V(b) + \boldsymbol{Q}]\} \\ \Delta_i(a,b) &= \{V_i(a) - [V_i(b) + q_i]\} \end{aligned} \qquad (12)$$

If a and b are not indifferent, instability can occur either if the preference is inverse or if it becomes an indifference in the modified complete semiorder.
In this case in which $\Delta(a,b) \neq 0$ (they are not indifferent), a condition for stability is:

$$\Delta(a,b)\,\Delta'(a,b) > 0.$$

It must be remembered that: Δ' is the equivalent of Δ for the V' and that

$$\Delta'(a,b) = \alpha\,\Delta(a,b) + (1-\alpha)\,\Delta_i(a,b)$$

Then, the stability condition becomes:

$$\Delta(a,b)\,\{\alpha\,\Delta(a,b) + (1-\alpha)\,\Delta_i(a,b)\} > 0 \quad \text{or}$$
$$\alpha\,\{\Delta(a,b)\,\Delta_i(a,b) - \Delta^2(a,b)\} < \Delta(a,b)\Delta_i(a,b) \qquad (13)$$

The last condition provides us with constraints on the parameter α ensuring stability for the pair a, b. It can be shown that the following situations hold:

i) If $\Delta(a,b)\,\Delta_i(a,b) > \Delta^2(a,b)$ (g_j is strongly in favor with the ranking of a and b), then (2) can be written as: $\alpha < \frac{\Delta(a,b)\,\Delta_i(a,b)}{\Delta(a,b)\,\Delta_i(a,b) - \Delta^2(a,b)} > 1$ giving an upper bound for the stability area;

ii) If $\Delta(a,b)\,\Delta_i(a,b) < 0$ (g_j disagrees with the ranking of a and b). In that case, we obtain from (2) the following expression: $\alpha > \frac{\Delta(a,b)\,\Delta_i(a,b)}{\Delta(a,b)\,\Delta_i(a,b) - \Delta^2(a,b)} < 1$, giving a lower bound for the stability area;

iii) If $0 \leq \Delta(a,b)\,\Delta_i(a,b) \leq \Delta^2(a,b)$, it is not possible that and inversion in the sense (1) could appear;

iv) If a and b are indifferent, $\Delta(a,b) = 0$, this indifference is maintained in the semiorder if and only if $\Delta'(a,b) = 0$.

It must be emphasized that the previous formulations have implicitly reflected the new preference structure introduced, taking into account an indifferent threshold whose values are included in the stability area of α parameter (Fernández & Escribano, 2006).

2.3 Different types of Stability

2.3.1. First Class: Full Stability

Full stability is defined as the absence of any modification in the whole *(P, I, R)* Semiorder Preference Structure.

To preserve the Semiorder in the case of Full Stability, no pair a, b of alternatives may verify (1), so that for all a, b in A that are not indifferent, the condition (2) must hold.

Let us define:

$$\Omega^0 = \{(a,b) \in \; A\; X\; A\;, \text{s.t.}\, \Delta(a,b) \leq 0 \text{ and } \Delta_i(a,b) > 0\} \qquad (14)$$
$$\Omega^0 = \{(a,b) \; \in A\; X\; A, \text{s.t.}\, [V(a) - V(b)] \leq \; \boldsymbol{Q} \text{ and } [V_i(a) - V_i(b)] > q_i\}$$

Ω^0 is the subset of all ordered pairwise of A which fulfill the constraint "if alternatives are indifferent under all criteria, there is at least one criterion g_i in which preference is preserved". Noticed that indifference threshold put is conditioning subset Ω^0.

$$\Omega^- = \{(a,b) \in A\; X\; A, \text{s.t.}\, \Delta(a,b)\; \Delta_i(a,b) < 0\;\} \quad (15)$$
$$\Omega^- = \{(a,b) \in A\; X\; A, \text{s.t. } [V(a) - V(b) - \boldsymbol{Q}]\; [V_i(a) - V_i(b) - q_i] < 0\}$$

Ω^- is the subset of all ordered pairwise subsets of A that fulfill the inversed preference, that is, those in which a change in opposite sense have been produced in preferences.

$$\Omega^+ = \{(a,b) \in A\; X\; A, \text{s.t.}\, \Delta(a,b)\; \Delta_i(a,b) > \Delta^2(a,b)\} \quad (16)$$
$$\Omega^+ = \{(a,b) \in A\; X\; A, \text{s.t.}\, [V(a) - V(b) - \boldsymbol{Q}]\; [V_i(a) - V_i(b) - q_i] > \; [V(a) - V(b) - \boldsymbol{Q}]^2\}$$

Ω^+ is the subset of all ordered pairwise subsets of A for which the initial ordering is kept, that is Strict Preference even when changes in weights of g_i criterion are allowed but keeping constant the rest.

The limits of α Stability Interval will be obtained in the following way:

$$\alpha_i^- = \max_{(a,b)\,\in\,\Omega^-} \frac{\Delta(a,b)\Delta_i(a,b)}{\Delta(a,b)\Delta_i(a,b) - \Delta^2(a,b)} \quad (17)$$
$$\alpha_i^+ = \min_{(a,b)\,\in\,\Omega^+} \frac{\Delta(a,b)\Delta_i(a,b)}{\Delta(a,b)\Delta_i(a,b) - \Delta^2(a,b)} \quad (18)$$

Then the stability of the semiorder is obtained for: $\alpha_i^- < \alpha < \alpha_i^+$ if Ω^0 is empty. If $\Omega^0 \neq \emptyset$, then the stability area for α is reduced to the only value 1 and no change of weights is allowed because any change will modify the preference structure, indifference becomes in preference.

A similar interval could be constructed for β. It is an easy task due to the direct relationship between α and β:

$$\beta = \frac{1 - w_i}{w_i}(1 - \alpha) \quad (19)$$

The stability area is: $\beta_i^- < \beta < \beta_i^+$, with:

$$\beta_i^- = \frac{1 - w_i}{w_i}(1 - \alpha_i^+) \quad (20)$$
$$\beta_i^+ = \frac{1 - w_i}{w_i}(1 - \alpha_i^-) \quad (21)$$

Finally, the NWSI for the weight of criterion g_i is defined as: (w_i^-, w_i^+), with:

$$w_i^- = (1 + \beta_i^-)\, w_i \quad (22)$$
$$w_i^+ = (1 + \beta_i^+)\, w_i \quad (23).$$

2.3.2. Second Class: Partial Stability

In that case the decision-maker is only interested in the stability of a subset *N* of the set *A* of all the feasible alternatives; that is, the stability of only a part of the preference structure. It can be defined as the absence of any change in the restriction of *(P, I, R)* gives by *N X A*.

The sets Ω^0, Ω^- and Ω^+ previously defined are replaced by:

$$\Omega_N^0 = \{(a,b) \in N\,X\,A \text{ , s.t. } \Delta(a,b) \leq 0 \text{ and } \Delta_i(a,b) > 0\} \quad (24)$$
$$\Omega_N^0 = \{(a,b) \in N\,X\,A, \text{ s.t. } [V(a) - V(b)] \leq \boldsymbol{Q} \text{ and } [V_i(a) - V_i(b)] > q_i\}$$

$$\Omega_N^- = \{(a,b) \in N\,X\,A, \text{ s.t. } \Delta(a,b)\, \Delta_i(a,b) < 0\} \quad (25)$$
$$\Omega_N^- = \{(a,b) \in N\,X\,A, \text{ s.t. } [V(a) - V(b) - \boldsymbol{Q}]\, [V_i(a) - V_i(b) - q_i] < 0\}$$

$$\Omega_N^+ = \{(a,b) \in N\,X\,A, \text{ s.t. } \Delta(a,b)\, \Delta_i(a,b) > \Delta^2(a,b)\} \quad (26)$$

$$\Omega_N^+ = \{(a,b) \in N\ X\ A, \text{s.t.}\ [V(a) - V(b) - \boldsymbol{Q}]\ [V_i(a) - V_i(b) - q_i] > [V(a) - V(b) - \boldsymbol{Q}]^2\}$$

This results generally in a wider NWSI and inside it the semiorder preference structure will be preserved in the selected subset.

2.3.3. Third Class: Set of "Good Alternatives" Stability

This case appears when the decision maker wishes to eliminate the worst alternatives of *A* and to obtain a subset of "good alternatives". For that reason, the stability of a subset is considered as the stability of set *G*.

In PROMETHEE II, the set *G* of good alternatives will be the following:

$G = \{a \in A, \text{s.t.}\ \phi(a) > \boldsymbol{Q}\}$, taking into account that: $\phi(a) = \phi^+(a) - [\phi^-(a) + \boldsymbol{Q}]$ because the preference structure we are studying is a Semiorder.

In that way, we have that:

$$G = \{a \in A, \text{s.t.}\ V(a) > \boldsymbol{Q}\}$$

and the condition for preference inversed is:

$$V(a)V'(a) < 0$$
$$[V(a) - \boldsymbol{Q}]\ [V'(a) - \boldsymbol{Q}] < 0$$

The constraints on the α parameter are given by the following expression:

$$\alpha\,\{[V(a) - \boldsymbol{Q}]\ [V_i(a) - q_i] - [V(a) - \boldsymbol{Q}]^2\} > \{[V(a) - \boldsymbol{Q}]\ [V_i(a)] - q_i\} \quad (28)$$

The limits of α can be obtained by a slight modification of the developed to full stability: Δ and Δ_i will be replaced by *V* and V_i , and the pairs of alternatives *(a, b)* by single alternatives $a \in A$.

3. Numerical Applications

The PROMETHEE Methods with NWSI in a Semiorder Preference Structure was applied to different problems in order to prove the most important findings of the new approach.

In Fernández G. (1991), different multicriteria decision problems have been solved using the PROMETHEE with its original WSI and have been explained in full. Moreover, the same problems have been managed with the PROMETHEE but under the Semiorder Preference Structure and the NWSI have played a special role.

In this paper we will present the most important results in relation with the NWSI and their comparisons with the WSI.

3.1. Problem 1: A Location Problem

For a complete description of the problem see Fernández (1991, pp. 307-331).

The problem relates to making a selection among six alternatives for the location of hydroelectric power plants using six decision criteria under a Semiorder Preference Structure and defining NWSI.

The decision matrix in Table 1 shows the evaluation of the alternatives and the type of generalized criteria associated with each original criteria:

Table 1. Decision matrix

Criteria / Alternatives	**g_1 Manpower**	**g_2 Power (MW)**	**g_3 Construction costs**	**g_4 Maintenance costs**	**g_5 Villages to evacuate**	**g_6 Security level**
A_1: Italy	80	90	60	5.4	8	5
A_2: Belgium	65	58	20	9.7	1	1
A_3: Germany	83	60	40	7.2	4	7
A_4: Sweden	40	80	100	7.5	7	10
A_5: Austria	52	72	60	2.0	3	8
A_6: France	9	96	70	3.6	5	6
Generalized Criteria:	Type II	Type III	Type V	Type IV	Type I	Type VI
q Parameter	10	-	5	1	-	-
P Parameter	-	30	45	5	-	-
σ Parameter	-	-	-	-	-	5

The PROMETHEE positive, negative and net flows calculated by DecisionLab software in the original version of the methodology are shown in Table 2.

Table 2. PROMETHEE flows (Preorder preference structure)

Alternatives	**Positive Flow**	**Order**	**Negative Flow**	**Order**	**Net Flow**	**Order**
A_1: Italy	0.222	6	0.366	5	-0.146	6
A_2: Belgium	0.396	2	0.379	6	0.017	2
A_3: Germany	0.247	5	0.336	2	-0.090	5
A_4: Sweden	0.329	3	0.349	3	-0.020	3
A_5: Austria	0.455	1	0.162	1	0.293	1
A_6: France	0.300	4	0.355	4	-0.055	4

The different flows obtained under the Semiorder Preference Structure are presented in Table 3:

Table 3. PROMETHEE flows (Semiorder preference structure)

Alternatives	Positive Flow	Order	Negative Flow	Order	Net Flow	Order
A_1: Italy	0.306	6	0.452	5	-0.232	6
A_2: Belgium	0.482	2	0.465	6	-0.069	2
A_3: Germany	0.333	5	0.422	2	-0.175	5
A_4: Sweden	0.415	3	0.435	3	-0.106	3
A_5: Austria	0.541	1	0.248	1	0.207	1
A_6: France	0.386	4	0.441	4	-0.141	4

The calculation process of the different flows is more difficult under the semiorder but the enrichment of the analysis allows to a better knowledge of the decision-maker preferences in the pairwise comparison of the alternatives. The decision-maker has an active participation and a harder interactivity during the full decision process.

Semiorder Preference Structure – Full Stability: The determination of the NWSI requires a lot of calculation during the full process. It is necessary to work with the net flows of each criterion particularly considered but taking into account new elements like the indifference threshold of each criterion and the Outranking Indifference Threshold. Of course, the use of specialized software makes the decision maker's task easy (Ríos & French, 1991).
The NWSI and the WSI are given below in Tables 4 and 5.

Table 4. Weight stability intervals

Criteria	Weight	Interval	%	Interval %
g_1: Manpower ($\$10^9$)	1.00	[0.90, 1.28]	16.67	[15.19, 20.37]
g_2: Power (MW)	1.00	[0.84, 1.27]	16.67	[14.40, 20.31]
g_3: Construction costs ($\$10^9$)	1.00	[0.86, 1.33]	16.67	[14.68, 21.06]
g_4: Maintenance costs ($\$10^6$)	1.00	[0.65, 1.35]	16.67	[11.56, 21.24]
g_5: Villages to evacuate	1.00	[0.86, 1.52]	16.67	[14.68, 23.31]
g_6: Security level	1.00	[0.37, 1.26]	16.67	[6.94, 20,10]

Table 5. New weight stability intervals

Criteria	Weight	Interval	%	Interval %
g_1: Manpower	1.00	[0.19, 1.54]	16.67	[0.66, 23.53]
g_2: Power (MW)	1.00	[0.76, 1.22]	16.67	[13.21, 19.56]
g_3: Construction costs ($\$10^9$)	1.00	[0.43, 1.51]	16.67	[7.89, 23.21]
g_4: Maintenance costs ($\$10^6$)	1.00	[0.50, 1.51]	16.67	[1.05, 23.23]
g_5: Villages to evacuate	1.00	[0.79, 1.78]	16.67	[13.72, 26.25]
g_6: Security level	1.00	[0.00, 5.00]	16.67	[0.00, 49.99]

At first, it can be noted that higher intervals belong to those criteria whose generalized associated criteria have threshold *q. That is, whose associated generalized criteria have* g_1*: Manpower (*q_1*=10);* g_3*: Constructions costs (*q_3 *=*

5); g_4: Maintenance costs (q_4= 1). In general, all the new intervals have lower inferior limits and higher superior limits than the original intervals (Le Teno; Mareschal, 1998).

It can be pointed out that the Semiorder Preference Structure is more stable than the Preorder one. The decision maker could express his preferences more openly in the "Freedom Decision Space". The sensitivity analysis could be done by changing the criteria weights and a final robustness study could be considered in order to guarantee the stability of the rankings obtained (Pseudo Robust; Perfectly Robust; Approximately Robust) (Roy, 1990; Vincke, 1989).

3.2. Problem 2: Car Selection

A full description of this problem is given in Fernández, G. (1991; pp. 331-344).

Car selection problem is a very common decision problem at present because there are a lot of trademarks that offer good prices and excellent quality of their products. The present problem has six alternatives (car model and trademark) which are evaluated under six decision criteria.

The decision matrix in Table 6 shows the evaluation of the alternatives and the type of generalized criteria associated with each original criteria:

Table 6. Decision matrix

Criteria / Alternatives	**g_1 Price**	**g_2 DIN Power**	**g_3 Fiscal Power**	**g_4 Maximum speed**	**g_5 Urban Consumption**	**g_6 90 km/h Consumption**
A_1: Quattro	251.00	9200	11	220	12	9.0
A_2: 6.35 Csi	263.24	218	20	229	15	11.0
A_3: 500 SEC	362.80	231	34	220	17	12.0
A_4: Porsche	231.80	204	16	235	13	8.5
A_5: HJS HE	267.00	295	31	2245	18	11.0
A_6: Esprit 3	239.00	162	11	220	14	8.0
A_7: GTBi	322.00	214	17	240	16	9.0
A_8: Jalpa	327.00	255	20	250	17	10.0
Generalized Criteria:	Type V	Type V	Type IV	Type VI	Type VI	Type VI
q Parameter	1	3	0.5	-	-	-
P Parameter	5	8	1.5	-	-	-
σ Parameter	-	-	-	5	0.5	0.5

The PROMETHEE positive, negative and net flows calculated by DecisionLab software in the original version of the methodology are presented in Table 7. The different flows obtained under the Semiorder Preference Structure are given in Table 8.

As this problem is generally the same as the previous problem, we will present its NWSI and the WSI and compare them with those of the previous problem:

Table 7. PROMETHEE flows (Preorder preference structure)

Alternatives	Positive Flow	Order	Negative Flow	Order	Net Flow	Order
A_1: Quattro	0.541	3	0.315	2	0.226	2
A_s: 6.35 Csi	0.382	7	0.503	6	-0.122	7
A_3: 500 SEC	0.140	8	0.771	8	-0.631	8
A_4: Porsche	0.639	1	0.250	1	0.388	1
A_5: XJS HE	0.408	6	0.516	7	-0.109	6
A_6: Esprit 53	0.547	2	0.349	3	0.198	3
$A_{7:}$ 308 GTBi	0.462	4	0.407	4	0.055	4
A_8: Jalpa	0.449	5	0.454	5	-0.006	5

Table 8. PROMETHEE flows (Semiorder preference structure)

Alternatives	Positive Flow	Order	Negative Flow	Order	Net Flow	Order
A_1: Quattro	0.550	3	0.324	2	0.217	2
A_s: 6.35 Csi	0.391	7	0.512	6	-0.131	7
A_3: 500 SEC	0.149	8	0.780	8	-0.640	8
A_4: Porsche	0.648	1	0.259	1	0.379	1
A_5: XJS HE	0.417	6	0.525	7	-0.118	6
A_6: Esprit 53	0.556	2	0.358	3	0.189	3
$A_{7:}$ 308 GTBi	0.471	4	0.416	4	0.046	4
A_8: Jalpa	0.458	5	0.463	5	-0.015	5

Table 9. Weight stability intervals

Criteria	Weight	Interval	%	Interval %
g_1: Price	1.00	[0.29, 1.41]	16.67	[5.52, 21.96]
g_2 DIN Power	1.00	[0.92, 1.49]	16.67	[15.50, 22.98]
g_3: Fiscal Power	1.00	[0.27, 1.19]	16.67	[5.11, 19.19]
g_4: Maximum speed	1.00	[0.90, 1.77]	16.67	[15.32, 26.12]
g_5: Urban Consumption	1.00	[0.70, 1.07]	16.67	[12.33, 17.66]
g_6: 90 km/h consumption	1.00	[0.29, 1.33]	16.67	[5.46, 20,97]

In general terms, all the NWSI under the Semiorder Preference Structure are wider than the WSI under the Preorder Preference Structure. This conclusion is the same, both in the case of normalized or no-normalized weights.

Table 10. New weight stability interval

Criteria	Weight	Interval	%	Interval %
g_1: Price	1.00	[0.00, 1.86]	16.67	[0.00, 27.17]
g_2 DIN Power	1.00	[0.38, 4.58]	16.67	[7.15, 47.80]
g_3: Fiscal Power	1.00	[0.14, 4.28]	16.67	[2.76, 46.13]
g_4: Maximum speed	1.00	[0.00, 3.38]	16.67	[0.00, 40.37]
g_5: Urban Consumption	1.00	[0.00, 12.29]	16.67	[0.00, 71.09]
g_6: 90 km/h consumption	1.00	[0.00, ∞]	16.67	[0.00, 100]

In addition, it can be pointed out that criteria g_1, g_4, g_5, g_6 have a NWSI with value zero in the inferior limit. For that reason these criteria could be eliminated from the analysis without changes in the total ranking, whichever it is the preference structure.

The extreme point is represented by criterion g_6 whose NWSI has the minimum allowed value in the low limit and the maximum allowed value in the high limit. Without any doubt, this criterion could be eliminated from the analysis without changes in the preferences.

Finally, a robustness analysis would be very important in order to conclude if our model is pseudo-robust, perfectly-robust or approximately-robust (Wolters & Mareschal, 1995).

4. Final Remarks and Conclusions

Finally, it should be pointed out that the "strong" stability (necessary and sufficient) condition for a pair *(a,b)* of alternatives that are not indifferent when the preference structure is a semiorder, namely:

$$\{V_p(a) - V_p(b) - \boldsymbol{Q}\}\{V_p'(a) - V_p'(b) - q_j'\} > 0 \qquad p = 1, \dots, r.$$

As it can be proved, under none point of view, the preference inversion is allowed. Not even in the case, in which a new indifference threshold is introduced, it would be possible to go from Preference to Indifference.

In that way, the NWSI could be obtained by applying the different formulas, depending on whether the decision maker wants a full stability, a partial stability or a set of good alternatives' stability. The limits in which stability is reached are the Weight Stability Interval for criterion g_i. Weights can have any value inside this interval in such a way that initial results are not modified (Mészáros & Rapcsák, 1996).

The contributions of the proposed NWSI under a Semiorder Preference Structure can be summarized as follows:

1. It gives the decision-maker more freedom to express his preferences and to modelize the situations in which he hesitates by the imprecision of the information he has.
2. It allows the decision maker to work with a robustness methodology that allows him to distinguish among different types of robustness situations (perfect, approximately or pseudo- robust).
3. It introduces new generalized criteria in PROMETHEE Methods. These new criteria are defined in concordance with the underlying preference structure (Fernández & Escribano, 2006). This is one of the most important contributions as it combines more complex preference structures with generalized criteria with more than two thresholds. In that case it can distinguish among semi criteria; pseudo criteria; order the intervals criteria and pre criteria. A full description of the new generalized

criteria and some real application can be seen in (Fernández & Escribano, 2006; Fernández et al., 2009; Fernández et al., 2011; Fernández et al., 2012).

4. The way to consider the aggregating function and the decision maker's preference make the difference between PROMETHEE modified methods and other MCDM Methods as TOPSIS, ELECTRE or VIKOR. In the previous methodologies, no distinction is made in the definition of weight stability intervals and their relationship with the underlying preference structure.

One of the most recent contributions to the study of stability analysis in MCDM is presented in the extended VIKOR Method (Opricovic & Tzeng, 2007). This method was developed to solve MCDM problems and focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. As the optimal solution does not exist it proposes compromise solutions. An extension of this method is a stability analysis determining weight stability intervals and with trade-offs analysis.

Both PROMETHEE and VIKOR methods are considered effective tools in multicriteria decision aid, particularly in situations where the decision-maker is not able or does not know how to express his/her preference at the beginning of the decision process. However, they differ in the determination of the weight stability intervals using different procedures.

We have just studied the determination of NWSI in PROMETHEE Methods considered as an additive method (section 2.1). However, the VIKOR method does not belong to this class of methods. This method introduces trade-offs on connection with a linear normalization, assuming the decision-maker is willing to approve these trade-offs. VIKOR method assumes the existence of linear relationship between each criterion function (generalized criteria in PROMETHEE Method) and a decision-maker's utility. In the contrary, in PROMETHEE Methods this relationship is not necessarily linear because it depends on the preference structure we are working with. In this case, it is important to remember that the generalized criteria must be defined in concordance with the underlying preference structure (new generalized criteria different from the six traditional types considered by PROMETHEE). Moreover, PROMETHEE is based on the maximum of group utility, whereas the VIKOR method not only considers the maximum of group utility but the minimal individual regret as well.

To decide which method to apply is not an easy decision because more advantages of one of them are compensated with less drawbacks of the other. For each problem we need to validate the decision making procedures and to evaluate the consequences of the application feasibility.

Without any doubt, studying weights sensibility in PROMETHEE Methods is enriched by a Semiorder Preference Structure. The Semiorder Preference Structure improves the analysis and the results given, not only by these methods but also by the Geometrical Analysis for Interactive Aid (GAIA), the Visual Modelization Technique associated to PROMETHEE (Mareschal, 1988a).

Much time would be saved and even more participation would be given to decision maker to the extent that he would be able to denote the type of the whished stability.

The main objective of the NWSI formulation is to have better techniques to do weight sensitivity analysis and even to save time in this tedious task. They also allow to a detailed knowledge of the decision problem.

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