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Enterprise Formation by Pooling: The Case of Ghana

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Abstract

The study examines the formation of an enterprise through the pooling of the resources of a group of individuals over a period of time, as opposed to the conventional approach of seeking institutional loans. Mathematical models and/or mathematical expressions/formulas are developed for the managements of the pooled funds under a general case and under some special cases. The developments of most of the mathematical expressions/formulas are based on the developments and proofs of some propositions. Mathematical analysis for determining the sustainability or viability of the enterprise or pooling arrangement is presented. Many of the models are illustrated using audited data from the largest shopping mall in Ghana. Some illustrations are also done using some hypothetical data. The illustrations done with the real world and the hypothetical data show that enterprise formation by pooling is a good and viable source of funding for the establishments of small-and medium-scale enterprises.

Keywords: *acquisition period, Andoh's inequality, crowd funding, honesty test inequality, strain, shadow amount.*

1. Introduction

This study is predicated on the belief that crowdfunding possess the potential to become an alternative source of financing for small and medium-scale businesses (SMEs) in Ghana. Capital is the lifeblood of any business operation. Consequently, sourcing capital to invest into business operations is a major requirement for enterprises, particularly SMEs. Nevertheless, traditional funding sources, in Ghana as in most developing economies, pose several challenges for SMEs.

One alternative financing scheme used in most developing countries as a source of finance for start-ups is rotational savings, popularly known in Ghana as "susu". Notwithstanding its usefulness in providing the beneficiaries with funds that they otherwise would not have, the micro contributions require from its members makes it ideal for those income-generating activities that reside in the informal sector of the economy as opposed to small enterprises that operates in the Ghanaian formal economy, which form the focus of this study. It is also

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tempting to consider philanthropic sources as possible sources of fund for Ghanaian small enterprises. However, philanthropic sources are largely suited for socially desirable projects as opposed to profit-oriented businesses.

Many microfinance institutions (MFIs) have sprung up in Ghana offering start-up funds to enterprises that have difficulty acquiring loans in the formal banking sector. However, recent global developments in MFIs show that aggressive commercialization is accompanied by poor lending approaches which results in the provision of credits to borrowers irrespective of their levels of indebtedness (Norwegian Microfinance Initiative, 2010). The mode of loan recovery from clients, the transfer of administrative cost of loan recovery to the poor and high rate of interest charged by these MFI's are among the major controversies facing the MFIs industry.

Using traditional sources such as soliciting loans from banks can be daunting and expensive due to high interest rates, which is currently running at 27 percent in Ghana. Also, banks tend to be very aware of the risks associated with SMEs. Mainstream banks may also not be "friendly" to SME owners, particularly the owners of new firms that want to expand their products, build new facilities, or develop markets (Bruns and Fletcher, 2008). Moreover, banks require collateral that can exceed the value of the initial loan by two-to-three times and in situations where SMEs are experiencing difficulties, banks may withdraw their assistance.

Again, venture capital funds mainly focus on the provision of risk capital to firms that have above average growth prospects, are profitable, and have a complete management team. They traditionally provide financing to less than 10 percent of SMEs that require finance (Klonowski, 2012; citing Oakey, 2007). They are highly selective in their choice of investee firms and may even place significant impositions onto a business owner. Venture capitalists focus on exit, which aims to provide them with an opportunity to cash out of their illiquid investment; this may force the investee firm to be sold to strategic investors. Venture capitalists may also take an active role in approving key operational, strategic, and financial decisions, which can be disruptive to business owners who value their independence.

These difficulties have imposed a pattern of constraint on Ghanaian SMEs which is not consistent with conditions needed for the growth of small enterprises. It is therefore not surprising that various studies on Ghanaian small enterprises, including Ahinful (2012), Abor & Biekpe (2007), Abor & Biekpe (2006), Mensah (2004) and Gallardo (2001) have highlighted the constraint that small enterprises face in their bid to access credit from mainstream financial institutions.

These underscore the need to have alternative funding sources for enterprises, especially start-ups. Consequently, we circumvent these conventional approaches of sourcing capital by mopping up the excess resources of a group of individuals over a period of time and channeling it into the establishment of a new business, thereby automatically transforming the individuals into shareholders of the business. This approach allows any business to access capital that it would otherwise lack.

Nonetheless, there are some serious questions to which many participants may need answers: (1) For how long will contribution last and how much can be realized for participating in the pool? (2) What happens to a participant's investment if the enterprise is a fiasco? (3) Can a participant pull out of the arrangement or join at a later date? What happens to a participant's investment if one can no longer contribute? Among others, these are questions that the study seeks to address.

The paper is organized as follows: section 2 deals with literature review. In reviewing the literature, we examined the few literature and/or materials we could find in the public domain on crowd funding. In section 3, we develop mathematical models for managing the pooled funds and the enterprises when they are operational. Some propositions and their proofs are also presented in the section. Some analysis and models for determining the sustainability or viability of the enterprise or pooling arrangement are presented in section 4. Section 5 uses real world empirical data and hypothetical data to illustrate the mathematical models developed in section 3 and the concepts of crowd funding. Conclusions made from the research are presented in section 6.

2. Literature Review

The concept of crowd-funding has its root in the broader concept of crowdsourcing, which refers to using the crowd to obtain ideas, feedback and solutions in order to develop corporate activities (Kleemann et al., 2008). In the case of crowd-funding, the objective is to collect money for investment. In other words, instead of raising the money from a very small group of 'professional' investors as in partnership, the idea of crowd-funding is to obtain it from a large pool (the crowd), where each individual will provide a very small amount. This can take the form of equity purchase, loan, donation or pre-ordering of the product to be produced. The number of investors that can participate in crowdfunding in Ghana may exceed 20.

Crowd funding has legal safe-guards that protect the interests of the members of the scheme (Made, 2004). Crowd funded enterprise must be registered like any other enterprise operating in the formal economy. As a result, it automatically gains protection by virtue of becoming a legal entity. Consequently, the interests and assets of the crowdfunders are safe-guarded in the same way that the interests and assets of the investors participating in a partnership are safe-guarded (Made, 2004). Conversely, should the crowdfunders decide to recruit professionals from outside to manage the business or nominate some of their members (perhaps the entrepreneurs among them) as directors, those recruited may adopt the same principles of business management and legal protection as any other legally registered business. It follows from this; therefore, that crowdfunding offers legal protection to its funders.

However, Bradford (2012) argues that towards offering protection to funders, crowdfunding can be so expensive for the small offerings that it facilitates thus negating its usefulness as financial source for small entrepreneurs. Thus, Bradford (2012) laments that registration requirements for crowdfunders in the United States is prohibitively expensive (US$250,000 – US$500,000) for small enterprises. Consequently, Bradford proposes exemption for crowdfunders. Bradford's proposition therefore suits the Ghanaian situation where crowdfunding does not face registration restrictions under security laws. Nonetheless, perhaps in future when crowdfunding is likely to be popular in Ghana, the government may consider a careful examination of the crowdfunding regulatory offerings in the United States. The United States experience will at least, offer a structured framework that Ghana could draw lessons from.

In line with the bid by enterprises to raise capital, Anand (2003), argues that unlike governments that raises capital by offering securities to the public through a Direct Public Offering (DPO), corporations have generally not adopted DPO as a means of financing their business activities. Anand concludes that corporations could benefit greatly from conducting DPO's. Although Anand's study focuses on corporations rather than small enterprises that largely form the beneficiaries of crowdfunding, to the extent that both small enterprises and corporations constitute businesses, and as such require capital, the concept of DPO is applicable to crowdfunding. It s therefore little surprise that crowdfunding operates on the principle of DPO.

While the popularity and significance of crowdfunding as an alternative source of external financing for businesses, particularly SMEs, continues to grow, there is still lack of unanimity as to how the concept should be defined. In view of this, several attempts have been made by researchers to define what crowd-funding is. For instance, Rubinton & Errunza (2011) defined crowd-funding as the process of one party financing a project by requesting and receiving small contributions from many parties in exchange for a form of value to those parties. Similarly, Belleflamme et al. (2011) viewed crowd-funding as one party's attempt to finance a project by offering three types of investment opportunities to potential investors. The investment opportunities are donations, passive investments, and active investments.

Ordanini et al. (2011) also assert that crowd-funding is an initiative undertaken to raise money for a new project proposed by someone, by collecting small to medium-size investments from several other people (i.e. a crowd). Belleflamme et al. (2011) further observed that crowd-funding involves an open call, mostly through the internet, for the provision of financial resources either in the form of donation or in exchange for the future product or some form of reward and/or voting rights (ibid). Although there seems to be lack of unanimity on its definition, the basic idea of crowd-funding is to raise external finance from a large audience (the "crowd"), where each individual provides a very small amount, instead of soliciting a small group of sophisticated investors (Belleflamme et al., 2011).

Despite the potential of crowdfunding to alleviate the constraint that Ghanaian small enterprise operators face, it has to be said that hardly has any research been carried out on crowdfunding in Ghana. The reason for this is that crowdfunding is an emerging phenomenon. Hence, very little has been written on the concept even in the United States where it originated from. Rather, most literature on Ghanaian small enterprises largely focuses on the constraints that hinder their growth.

3. Model Developments, Propositions and Proofs

In this section, we develop some mathematical models/expressions for managing the pooled funds and the established enterprises when they are operational. We also develop some propositions and establish their proofs.

3.1. General Model Defining Investors' Total Contribution

Suppose a group of κ people wish to establish an enterprise at a price T_f by the end of time N by pooling resources together. The amount T_f includes compensation to contributors for providing the stream of cashflows. We call N the acquisition period. Suppose that at the beginning of time $j, j = 1,..., N,$ each participant $i, i = 1,..., \kappa,$ contribute $s_{f_{ij}}$. Suppose also that the amount collected at each time $j, j = 1,..., N$, is invested until the enterprise is established at the end N of the time sequence (i.e. at time $j = N$). Then at time $j,$ the amount collected and invested will be $\sum_{i=1}^{\kappa} s_{f_{ij}}$. When the enterprise is established, $\sum_{j=1}^{N}\sum_{i=1}^{\kappa} s_{f_{ij}}$ would have been collected and invested in the business. Let r be the continuous rate of interest in the economy. Accounting for the time value of money of contributors, the total amount collected up to time N by the κ people is given by

$$\sum_{j=1}^{N}\sum_{i=1}^{\kappa} s_{f_{ij}} (1 + \mathrm{r})^{\mathrm{N-j}}$$

This amount gathered by time N should be sufficient to acquire T_f. Hence

$$\sum_{j=1}^{N}\sum_{i=1}^{\kappa} s_{f_{ij}} (1 + \mathrm{r})^{\mathrm{N-j}} = T_f \quad (1)$$

In the general case, we do not put restriction on the amount that participants contribute to the pool. We also permit late entry to the pool in the general case. A late entrance contribution can be considered as initial zero payments until the first payment is made at the time of entry. This allowance has the potential to reduce the enterprise acquisition period, N considerably. In addition, we will permit free exit on a restricted basis so as not to unduly jeopardize the acquisition period. The time taken to acquire the enterprise is nonlinear and has to be solved numerically (see for example Brandimarte (2002), pgs 111-117).

3.2. Propositions and Proofs

Proposition 1: For all $r \in [0,1]$,

$$\sum_{j=1}^{N}\sum_{i=1}^{\kappa} s_{f_{ij}} \leq T_f \leq \sum_{j=1}^{N}\sum_{i=1}^{\kappa} s_{f_{ij}}\, 2^{N-j}$$

for $j = 1,\dots, N.$

Proof: Because $0 \leq r \leq 1$ it follows that $1 \leq (1+r) \leq 2$. Hence

$$1 \leq (1+r)^{N-j} \leq 2^{N-j} \tag{2}$$

for all $j = 1,\dots, N.$ The result then follows by multiplying each term of the inequality in (2) by $\sum_{j=1}^{N}\sum_{i=1}^{\kappa} s_{f_{ij}}$ and using the equation in (1).

Proposition 1 places a cap on how much contributors can demand on the business within the acquisition period, and thereby protecting the enterprise from exploitation by investors when the business is up and running. If you consider any enterprise as a living organism with capital being its lifeblood, any unnecessary demands by owners can stifle its growth and the needs of other stakeholders whose lives depend on the enterprise. Proposition 1 is there to prevent this from happening (i.e. it gives a guideline as to the maximum amount investors or participants of the pool can ask of their investment).

Proposition 2: For each $i = 1,\dots,\kappa; j = 1,\dots, N$ and $s_{f_{ij}} \geq 0$

$$\sum_{j=1}^{N}\sum_{i=1}^{\kappa} s_{f_{ij}} \leq \frac{1}{Nr}\sum_{j=1}^{N}\sum_{i=1}^{\kappa} s_{f_{ij}}[(1+r)^N - 1] \tag{3}$$

where $r \in (0,1], N \in \mathrm{N}.$

The expression on the left-hand side (LHS) of the inequality sign in (3) represent the real amount invested in the enterprise when the enterprise is established. The expression on the right-hand side (RHS) can be considered the invested amount incorporating the time value of money. It can be considered as the cost of the enterprise from the investor's perspective.

The proof of the result follows from the following result.

Proposition 3 (Andoh's inequality)**:** For $N \in \mathrm{N}$ and $r \in (0,1]$,

$$Nr \leq (1+r)^N - 1$$

Proposition 3 can be viewed as saying that simple interest is less or equal to compounding interest.

Proof of proposition 2

The proof of proposition 2 follows directly from the proposition 3 by noting that $\sum_{j=1}^{N}\sum_{i=1}^{\kappa} s_{f_{ij}} \geq 0$ and $Nr \leq (1+r)^N - 1$.

A measure of the reward for investors at any time before the acquisition period can be expressed as

$$\frac{1}{Nr}\sum_{j=1}^{N}\sum_{i=1}^{\kappa} s_{f_{ij}}[(1+r)^N - 1 - Nr]$$

We call this the shadow amount! It is the amount investors are demanding on the enterprise at time $j = 1,..., N$ as a compensation for providing the stream of funds. It can also be viewed as the amount that attracts investors to participate in the pool.

In the formative period of any enterprise, owners generally have to make sacrifices for the enterprise to stand on its feet. The shadow amount incorporates this behavior of owners by not offering the full compounded amount in the formative period. Of course, if the enterprise does very well at the recouping stage, owners are also beneficiaries of profits that would be realized.

The time of the reward will depend on the time of entry to the pool. Irrespective of the time of entry, each participant can be viewed as entering the same time. A late entrance contribution can be considered as initial zero entry until the period of entry by managers of the fund. In the recouping stage (which we assume exceeds the formative period) there is a "waiting" period equivalent to the period of delay of entry to the pool.

Observe that as

$$\lim_{r\to 0} \frac{(1+r)^N - 1 - Nr}{r} = 0,$$

the shadow amount should lie in the interval

$$\left(0, \frac{(2^N - N - 1)}{N} \sum_{j=1}^{N} \sum_{i=1}^{\kappa} s_{f_{ij}}\right]$$

3.3. Some special cases and some more propositions

Here we consider some special cases and develop relevant mathematical expressions or formulas – via propositions - for applications under each case.

3.3.1. Case When all Funds Needed for the Establishment of the Enterprise are Available at Time $j = 1$

When all the funds needed for the establishment of the enterprise are available at time 1, contributors will have to be compensated for the time period between when contributions are made and the time the enterprise is completed N'. In such a case,

$$(1+r)^{N'} \sum_{i=1}^{\kappa} s_{f_i} = T_f$$

and

$$N' = \frac{\log\left(\dfrac{T_f}{\sum_{i=1}^{\kappa} s_{f_i}}\right)}{\log(1+r)}$$

Proposition 4: If $s_{f_{ij}} = s_f$ for each $i = 1,...,\kappa$ and each j, (i.e. each investor agree to pay the same every period) then

$$\frac{\log\left(\dfrac{T_f}{\kappa}\dfrac{1}{s_f} + 1\right)}{\log 2} \le N \le \frac{T_f}{\kappa}\frac{1}{s_f} \qquad (4)$$

Proof: If $s_{f_{ij}} = s_f$, then from proposition 1,

$$N\kappa s_f \leq T_f \leq \kappa s_f \underbrace{\sum_{j=1}^{N} 2^{N-j}}_{2^N - 1} \tag{5}$$

Taking the right side of (5), $T_f \leq \kappa s_f (2^N - 1)$, we have

$$\frac{\log\left(\frac{T_f}{\kappa}\frac{1}{s_f} + 1\right)}{\log 2} \leq N$$

Taking the left side of (5), $N\kappa s_f \leq T_f$, we get

$$N \leq \frac{T_f}{\kappa}\frac{1}{s_f}$$

Hence the desired result follows by combining these inequalities.

Remark 1: It follows from equation (1) that if $s_{f_{ij}} = s_f$ for each $i = 1,\ldots,\kappa$ and each j, then the exact time of enterprise completion is

$$N = \frac{\log\left(1 + \frac{T_f}{\kappa}\frac{1}{s_f}\right)}{\log(1+r)}$$

Compare with (Andoh, 2008) pg. 103. It should be noted that $\frac{T_f}{\kappa} \neq s_f$ as T_f includes compensation for investors for providing the stream of cashflows. In fact, $\frac{T_f}{\kappa} > s_f$!

Proposition 4 emphasizes the point that if all investors agree on paying equal amount, then the acquisition period, N, should not exceed the average stake in the enterprise multiplied by the reciprocal of the agreed amount. It gives investors the maximum waiting time for which recouping of funds should start and offers managers of the fund a time bound for which the enterprise has to be completed.

We call the inequality in (4) the "honest test inequality" as it gives indication to an investor the maximum waiting time for recouping of investments.

It is clear from the inequality (4) that as the number of contributors increase, the acquisition period decreases for fixed s_f. It should also be clear from proposition 4 that for a fixed number of people in the pool, the acquisition period decreases as the amount contributed by the people in the pool increases.

Proposition 5: If $s_{f\,ij} \in U[\alpha,\beta]$ (i.e. contributions are uniformly distributed in the interval $[\alpha,\beta]$), then the amount of funds invested in the enterprise at time $j=1,\ldots,N$ can be approximated by $\kappa\left(\frac{\alpha+\beta}{2}\right)$ as $\kappa\to\infty$ with the acquisition period N satisfying the inequality

$$\frac{\log\left(\frac{T_f}{\kappa}\frac{2}{(\alpha+\beta)}+1\right)}{\log 2} \le N \le \frac{T_f}{\kappa}\frac{2}{(\alpha+\beta)}$$

Proof If $s_{f\,ij} \in U[\alpha,\beta]$, then $E(s_{f_{ij}}) = \frac{\alpha+\beta}{2}$. It follows from the law of large numbers that for fixed j,

$$\sum_{i=1}^{\kappa} s_{f_{ij}} \to \kappa E(s_{f_{ij}})$$

as $\kappa\to\infty$.

The rest of the result follows from direct application of proposition 4.

Proposition 6: If $s_{f\,ij} = s_{f_i}$ for all j, (i.e. each contributor agrees to pay the same amount every time), then the total amount invested in the enterprise from the investors perspective is

$$\sum_{i=1}^{\kappa} s_{f_i} \frac{(1+r)^N - 1}{r}$$

where $r\in(0,1]$ and acquisition period N lies in

$$\left[\frac{\log\left(\dfrac{T_f}{\sum_{i=1}^{\kappa} s_{f_i}} \right)}{\log 2} + 1, \frac{T_f}{\sum_{i=1}^{\kappa} s_{f_i}} \right]$$

Proof If each person agrees to pay the same amount every time, then each investor $i = 1,...,\kappa$ contribution grows to $s_{f_i} \dfrac{(1+r)^N - 1}{r}$ at the acquisition period. Hence the total amount of funds invested from investors' perspective is

$$\sum_{i=1}^{\kappa} s_{f_i} \frac{(1+r)^N - 1}{r}$$

The rest of the proof follows by direct application of proposition 1.

3.4. Case Where Risk is Incorporated

The analysis so far assumes that everyone honors the contribution within the acquisition period. There is always the possibility that a major contributor may become incapacitated and for that matter could no longer contribute or even die before the acquisition period N. The shortfall in cashflow can be handled in several ways.

Insurance for the case of incapacitation or death: The risk of a contributor whose incapacitation or death can affect the acquisition period significantly can be transferred to an insurer. Insurance increases wealth when a loss occurs and decreases wealth when a loss does not occur. The increase in wealth when a loss occur can be viewed as the benefit of insurance and the reduction in wealth when a loss does not occur can be viewed as the cost of insurance (Harrington & Niehaus, 2004), pg 164.

To demonstrate how this works, let a premium P_j be paid at time $j = 1,...,N$. Let P_j be funded from the funds collected at time $j = 1,...,N$ by the κ people. Then the actual amount invested in the enterprise at time j will be $\sum_{i=1}^{\kappa} s_{ij} - P_j$. We assume that contributions from members at any time $j = 1,...,N$ far exceed the premium at that time (i.e. $\sum_{i=1}^{\kappa} s_{ij} >> P_j$ for all j). Consequently, at the enterprise completion, $\sum_{j=1}^{N}(\sum_{i=1}^{\kappa} s_{ij} - P_j)$ would have been invested. Accounting for the time value of money of contributors, we have the following result, similar to proposition 2.

Proposition 7: Let P_j be the premium payable at time $j = 1,..., N$. For each $i = 1,..., \kappa;\ j = 1,..., N$ and $s_{f_{ij}} \geq 0$,

$$\sum_{j=1}^{N}(\sum_{i=1}^{\kappa} s_{f_{ij}} - P_j) \leq \frac{1}{Nr}\sum_{j=1}^{N}(\sum_{i=1}^{\kappa} s_{f_{ij}} - P_j)[(1+r)^N - 1] \tag{6}$$

where $r \in (0,1], N \in \mathrm{N}$.

Proof: As $\sum_{i=1}^{\kappa} s_{f_{ij}} > P_j$ for all j, $\sum_{j=1}^{N}(\sum_{i=1}^{\kappa} s_{f_{ij}} - P_j) > 0$. The result follows from proposition 3.

The cost of insurance on the project will be $\sum_{j=1}^{N} P_j$, the total premium paid during the acquisition period N. The shadow amount in this case is

$$\frac{1}{Nr}\sum_{j=1}^{N}(\sum_{i=1}^{\kappa} s_{f_{ij}} - P_j)[(1+r)^N - 1 - Nr]$$

Of course P_j can be borne by members in the pool outside the amount collected at time j, in which case the real amount invested in the enterprise will be the LHS of proposition 2. In this case, additional cost to participants in the pool will be $w_i P_j, 0 < w_i \leq 1, \sum_{i=1}^{\kappa} w_i = 1$ where w_i is the fractional part of P_j that investor i has to contribute.

Retention: Members of the pool can decide to retain the risk and bear proportionally any shortcoming in the cash flow over the acquisition period. Alternatively, the shortfall in cash flow can be ignored and the remaining investors contribute until the enterprise is established. This option will take a bit longer period for the enterprise to be completed.

4. Sustainability and Pay back analysis (Recouping contributions)

Here in this section, we develop a mathematical expression for determining the profit that can be realized from the pooling arrangement at any time i. We also obtain mathematical expressions for the minimum number of products that should be produced by an enterprise and the minimum price per product unit for a pooling arrangement to be sustainable or profitable.

Suppose investors want to recoup their investments M years after the project has been acquired where $M > N$. Suppose also that the enterprise generates a periodic income $t_{r_i}, i = 1,2,\ldots,M$ at a total cost t_{c_i}. Then the profit realized at time i, P_{f_i} can be written

$$P_{f_i} = t_{r_i} - t_{c_i}$$

t_{c_i} can be written as

$$t_{c_i} = t_f + v_{c_i} + \tilde{T}_f \omega_i (1+r)^i,\ i = 1,2,\ldots,M$$

Here t_f and v_{c_i} are respectively the fixed and the variable cost of operations per period, $\omega_i, 0 < \omega_i \le 1,$ is the fractional part of $\tilde{T}_f$ to be paid at time $i = 1,2,\ldots,M$. $\tilde{T}_f$ is the true cost of the enterprise and might differ from T_f that was initially estimated as the cost of the enterprise. At the end time N, T_f will be known precisely and it will be called $\tilde{T}_f$.

The amount available at time $i = 1,2,\ldots,M$ to be shared among investors is

$$\tilde{T}_f \omega_i (1+r)^i, i = 1,2,\ldots,M$$

This amount acts as a strain on the business operations but it is an amount that is a bona fide property of investors. For viability at time $i = 1,2,\ldots,M$, $P_{f_i} > 0$. Hence

$$t_{r_i} > t_f + v_{c_i} + \tilde{T}_f \omega_i (1+r)^i, i = 1,2,\ldots,M$$

If t_{r_i} were obtained by selling a certain number of products n_p, at a unit price s_p, then the appropriate value for s_p, for sustainability of the pooling arrangement, is given by

$$s_p > \frac{t_f + v_{c_i} + \tilde{T}_f \omega_i (1+r)^i}{n_p}$$

On the other hand, if s_p is held fixed, then the number of products, n_p, to produce for viability of the pooling arrangement must be greater than

$$\frac{t_f + v_{c_i} + \tilde{T}_f \omega_i (1+r)^i}{s_p}$$

If $P_{f_i} \geq 0$ at any time i then, two sources of funds are available to the investor: a share in the profit and a portion of the periodic strain which belong to the investors.

5. Illustration of Models Using Hypothetical and Real World Examples

5.1. Hypothetical Example

Suppose an enterprise costing \$50000 is to be established and that all funds are available at time 1 (i.e. the first month). Let the rate of interest be 28% per annum (2.33% per month), the current rate of interest by banks. We investigate the effect of the shadow amount over time as the enterprise is under construction. It is clear from the Figure 1 (upper left) that a delay in getting the enterprise running at the shortest possible time is tantamount to draining out its “blood”. Thus it is paramount that managers of the fund get the enterprise running as quickly as possible as investors will have to be compensated higher for any delay. If, for example, the enterprise is up and running in the second month, investors have to be compensated for \$582.50. On the other hand, if the enterprise is up and running in the twelfth and twenty-fourth months, investors have to be compensated \$6932.24 and \$15994.86 respectively.

Next, suppose the cost of the enterprise incorporating compensation for investors is \$50000 and that everyone agrees to pay \$50 per month until the enterprise is up and running. Assume interest rate is 28% per annum (i.e. 2.33% per month). We first investigate how long it will take to get the enterprise established. Using the lower bound of the inequality (4), $N \geq \log_2\left(\frac{1000}{\kappa} + 1\right)$.

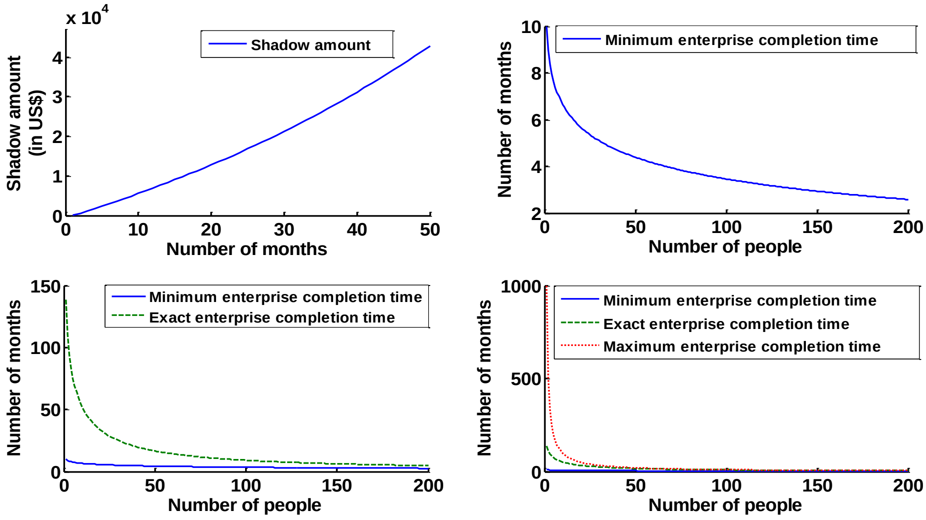

Figure 1. Upper left: Shadow amount with varying months**.** Upper right: Minimum time for project completion. Lower left: Minimum (solid line) and exact (dashed) time for project completion. Lower right: Minimum (solid line), exact (dashed line) and maximum (dotted line) time for project completion.

The minimum time for the enterprise to be completed is shown in the upper right of Figure 1. As the figure depicts, N decreases as κ increases. With 5 people in the pool, the minimum time for the enterprise to be up and running is about 7.7 months. With 10 people, the minimum time for enterprise completion is about 6.7; additional 5 people do not cause a greater gain in time for the enterprise to be on its feet. With 100 and 200 people in the pool, the minimum completion times for the enterprise to be completed are respectively about 3.5 and 2.6 months. The lower left plot is the exact and minimum time of enterprise completion. The lower right plots are the minimum, exact and the maximum enterprise completion times plotted together.

Next, suppose the minimum contribution is \$25 and the maximum contribution \$75 per month and that the contributions are uniformly distributed on the unit disk. We investigate the time taken for the enterprise to be acquired under two scenarios for 200 participants in the pool: first with increasing contributions and then with decreasing contributions.

It should be noted that the initial contributions for acquisition time with decreasing contributions are higher than the ones for acquisition times with decreasing contributions.

From remark 1, the time to acquire the asset, N , is given by:

$$N = \frac{\log\left(1 + \frac{rT_f}{\sum_{i=1}^{\kappa} s_{f_i}}\right)}{\log(1+r)}$$

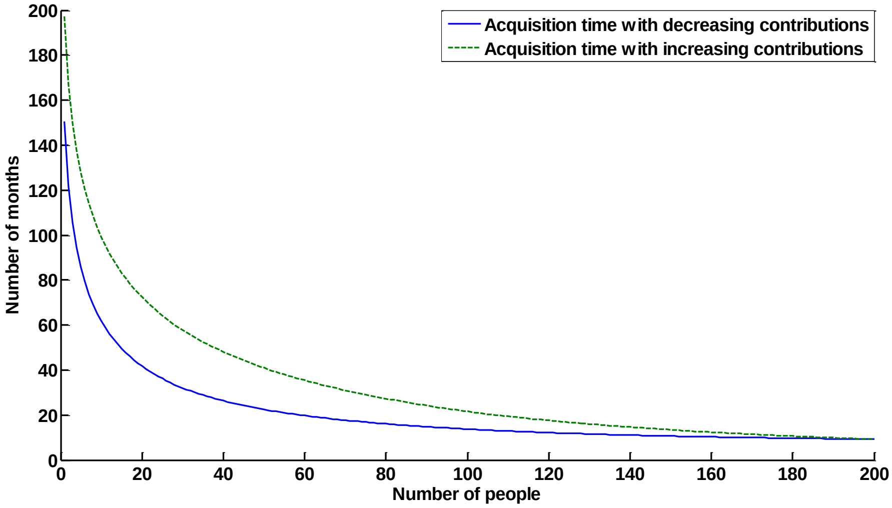

Figure 2. Acquisition time with increasing (dashed line) and decreasing (solid line) contributions

As can be seen from Figure 2, acquisition time with increasing contributions dominates the acquisition time with decreasing contributions. Thus the enterprise can be established at a faster rate when initial contributors' amounts are higher than when initial contributors' contributions are lower. In both cases, the acquisition time decreases at a decreasing rate as the number of people in the pool increases.

5.2. Illustrations with Real World Example/Empirical Data

Data for the illustrations here was obtained from the Accra Mall (the closest we could find that befits crowdfunding in Ghana). The construction of Accra mall was a partnership between a local entrepreneur (Joseph Owusu-Akyea), and Actis, a UK based private equity investor (http://www.accramall.com). Actis has 85% shares and the remaining 15% for the local entrepreneur. We could not obtain the composition of Actis and their holdings and as a typical equity firm comprises a large number of investors, we used the data obtained from the mall as a proxy for our illustrations.

The estimated cost of constructing the Accra mall is about 30 million dollars. The rentable space of the mall, excluding the car park, is 20000 square meters. These are partitioned into 62 shops paying an average of \$30 per square meter per month. The fixed costs of operation are electricity consumption, water and salary of workers which is estimated to be between \$160000 and \$165000 per month. The variable cost which are generally small are incurred in stationery, plumbing, maintenance, gardening (i.e. service charge) which is estimated to be \$20000 per month.

We assume that the strain on the mall will be paid over 25 years (300 months) and that the cost and revenue generation will not differ much within this period.

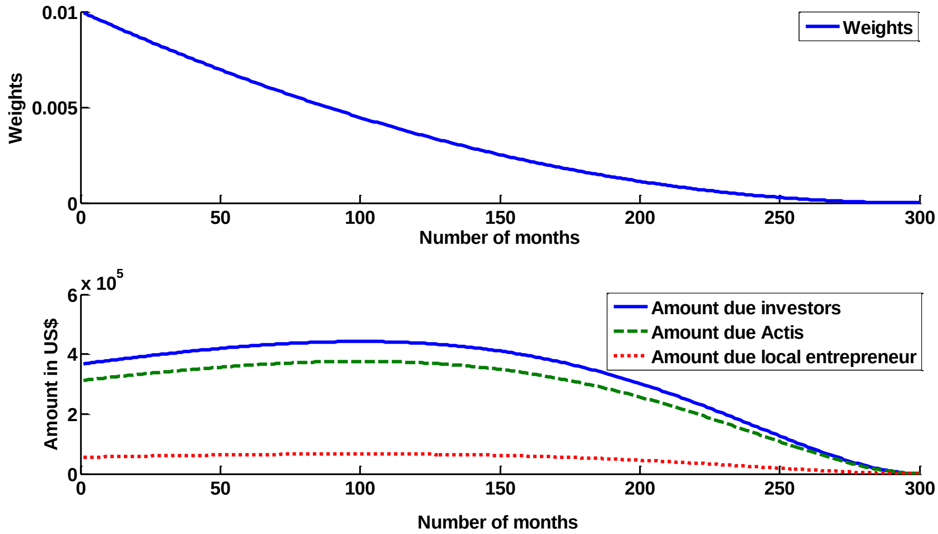

Figure 3. Upper plot: Weights applied to $\tilde{T}_f$. Lower plot: Amount of $\tilde{T}_f$ paid to investors over 25 years (solid), Actis share of $\tilde{T}_f$ over 25 years (dashed line) and the local entrepreneur share of $\tilde{T}_f$ over 25 years (dotted line).

Figure 3 (upper plot) shows the weights applied to $\tilde{T}_f$. The weights have decreasing compensation rate for investors for 300 months (25 years). In the first month for example a weight of 0.01 was applied whereas in the 300 month, a weight whose value is close to zero (i.e. $1.10564\text{x}10^{-7}$) was used. From the enterprise perspective, it is good to compensate investors more in early years than later years because of the expression $(1+r)^i, i=1,\ldots,N$ multiplying $\tilde{T}_f$ for an appropriate rate of interest r. A delay will mean that the enterprise will pay greater amount that will have negative impact on profitability in later years. Of course, most investor will want to be compensated more now than later and so the chosen weighting style is good for both the investor and the enterprise.

Figure 3 (lower plot) show the amount of $\tilde{T}_f$ paid to investors over 25 years (solid). The amount to be paid investors rises until it reaches the optimal value \$442249.40 at the 100 month and declines fast towards the 300th month. The corresponding profit for the 25 years for all stakeholders is depicted in figure 4 (upper plot).

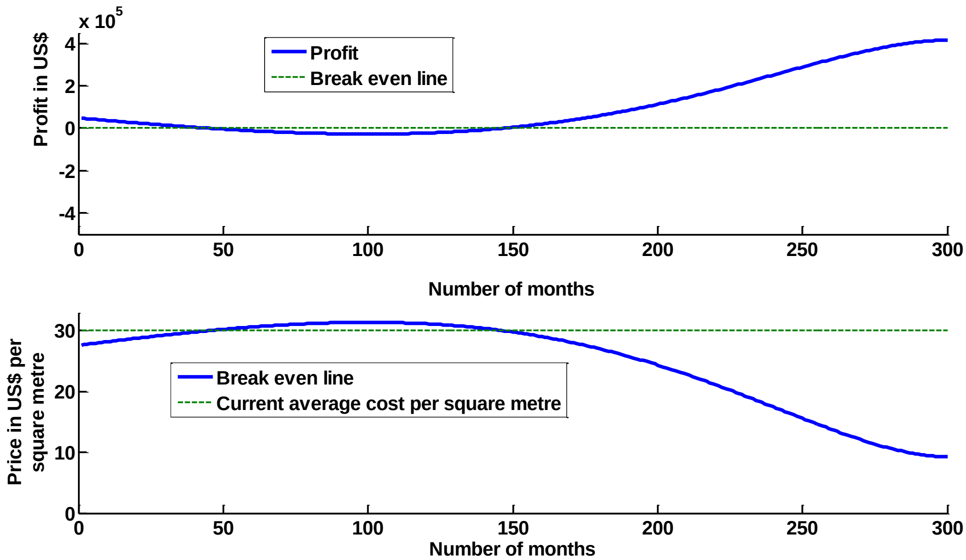

Figure 4. Upper plot: profit over 25 years. Lower plot: price in US dollars per square meter over 25 years.

As can be seen from the Figure 4 (upper plot), there is a downward dip of profits until around 45th month when profit becomes negative reaching its lowest point of -\$27249.40 around 100 month. There is then a gradual recovery of the company's profit breaking even around 147 month. Beyond this point, there is gradual increase in the company's profit until 300 month at which all $\tilde{T}_f$ would have been paid off. At its current price of \$30 per square meter per month the company is likely to experience negative profit in some years ahead.

5.3. Sensitivity analysis

In our sensitivity analyses, we seek to determine the price per square metre per month for which the mall enterprise will be profitable at all times at the current cost of operation.

To be profitable, the price per square metre per month, s_p must be such that

$$s_p > \frac{t_f + v_{c_i} + \tilde{T}_f \omega_i (1+r)^i}{n_p}$$

Substituting values for t_f, v_{c_i} and n_p as given earlier in section 5.2, we have that:

$$s_p > \frac{185000 + \tilde{T}_f \omega_i (1+r)^i}{20000}, i = 1,2,...,300$$

For the operations of the mall to be profitable at all times within the 25 year period, management has to set the price of the square meter of space to lie above the break even line (lower plot of Figure 4). The mall at its current price of \$30 dollars per month is profitable but must consider an upward revision beyond 3.7 years. The optimal point (which represents a price of \$31.36) was reached at the 100th month. Hence, if the price of square meter is set around \$31.50 per square meter per month, the operations of the mall will be profitable at all times.

Alternatively management can plough back the strain of investors in consultation with the investors within the 44th and 147th month so as to sustain the operations of the mall. A different weighting scheme that squeezes the amount of strain to be paid investors can also be adopted to ensure that the mall is profitable at all times.

6. Summaries and Conclusions

6.1. Summaries

In this study, we developed some mathematical models for managing pooled funds from a group of people for the formation of any enterprise and the established enterprises when they are operational. We derived expressions for the capital accumulation at any time, and an expression that allows participants know their stake in the enterprise any time. We also developed a mathematical expression for determining the profit that can be realized from the pooling arrangement at any time. In addition, we obtain mathematical expressions for the minimum number of products that should be produced by an enterprise and the minimum price per product unit for a pooling arrangement to be sustainable or profitable. Two sources of funds are available to the participant in any period where profit is realized: a share in the profit and a portion of the periodic strain.

6.2. Conclusions

The mathematical models we have developed should be useful to anyone interested in the formation of an enterprise by resource pooling. In particular it should assist the manager or the entrepreneur in the management of the pool of funds by contributors.

For such a scheme to succeed it is important that all funds collected from participants are judiciously invested in the project so as to motivate contributors to continue investing for the success of the scheme.

The fact that the acquisition period decreases at a decreasing rate as the number of people in the pool rises, raises an important question: at which point should admission to the pool be stopped? This will be left to the discretion of the managers of the fund and/or the entrepreneur.

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