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Public Private Partnership as an Alternative Source of Financing Highways in Ghana

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Abstract

This paper assesses the profitability of a private entity going into partnership with the Ghanaian government for the construction and management of highways. The paper derives the conditions under which public private partnership highway financing can be viable in Ghana. A mathematical model that can be used by a businessman or an organization (or a concessionaire) to determine the optimal profit, the optimal number of different vehicle sizes, and the optimal toll rates for a given concessionary period are developed. The model is a very good and useful planning and decision-making tool for any business man or organization interested in venturing into partnership with any government with regard to highway construction and management. Using the model and data from the Ghana Highway Authority, we show that public private partnership financing is applicable on a number of roads through the right mix of two variables, the concession period and the road toll rate. Our findings can be useful to investors interested in partnering with government in highway financing on the type of highway to choose and its accompanying cost. Furthermore, this paper will provide the government with better insight when partnering with the private sector in highway financing.

Keywords: *Concessionary period, mathematical model, optimization, reconstruction, rehabilitation, toll rate, upgrading.*

1. Introduction

Many years of underinvestment on the development of the highway infrastructure in Ghana have left the country with a significant infrastructure deficit which is holding back the development and economic growth of the country. Ghana's situation is no different from that of many developing countries on the African continent. Government in these countries needs to make massive investments from sources transcending public financing, in order to close this yawning infrastructure gap. Zitron (2004) defines a Public-Private Partnership (PPP) simply as "a long term relationship between the public sector and the private sector that has the purpose of producing public services or infrastructure". Although public private partnership (PPP) provides alternative source of financing that has proven to be workable worldwide, this credit source is yet to be deployed to resolve the infrastructure needs of these developing countries.

Ghana's case provides a good example. In line with measures to expand and improve the sources of road financing, the Government of Ghana introduced a number of restructuring programs in the road sector. The Road Fund was established in 1993 by Parliament Act 1245 of

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1996 to legalize the collection of road and bridge toll charges on some selected highways. Separate agencies with specific responsibilities for funding, road management and road safety, each reporting to different ministries were also established. Foreign donor-assisted programs such as the Highway Sector Investment Program (HSIP) Phase I, HSIP Phase II, Road Sector Program (RSP) and Road Sector Development Program (RSDP) were introduced at different phases for improving efficiency and effectiveness in road financing and management. Currentlythe Transport Sector Program (TSP) is supporting road development projects. The Road Fund received a total amount of about US$1.9 million from the new bridge and road toll charges collected from the existing 27 toll stations scattered throughout the country for the month of February, 2010 (Ghana Road Fund Secretariat, 2010).

Unfortunately, all these efforts have not resulted in achieving sustained and timely financing for managing the extensive trunk road network. Indications point to the fact that the current trend of funding levels from government to the highway sector will not change significantly. This is partly due to ever increasing competing demands on public funds from other important sectors of the national economy like education, health, water, housing and electricity.

Given the lack of adequate public financing, private financing under PPP arrangements can be tapped and relied upon to provide the additional funds needed on a sustainable basis to improve the general condition of the highway network in Ghana. Our contribution is to derive the conditions under which such an arrangement will work for both parties: the government and the private entity. It is worth mentioning that although this study is based on Ghana, its findings are applicable, at least in fundamental aspects, to developing countries in general, as Ghana is typical of these countries in that they share similar challenges regarding the need to make massive investments from sources beyond public financing, in order to close the yawning infrastructure gap.

The paper is organised as follows: section 2 deals with literature. There, we briefly review the literature and/or materials on past and current works on PPP arrangements in other countries. In section 3, we present our PPP model/analysis method. We also derive the conditions under which PPP arrangement can work. Given specified toll rates for different sized vehicles, we derive the number of each sized vehicle that can use the highway. Additionally, given the number of each sized vehicle that uses the highway, we derive the toll rate that should be charged for the PPP arrangement to be sustainable. Empirical results of analyses, based on data obtained from the Ghana Highway Authority, are presented in section 4. Conclusions and recommendations are presented in section 5.

2. Literature Review

PPP is a concession project. Smith (1999) defines a concession project as a project based on the granting of a concession by a principal, usually a government, to a promoter, sometimes known as the concessionaire, who is responsible for the construction, financing, operation and maintenance of a facility over the period of the concession before transferring, at no cost to the principal, a fully operational facility. During the concession period the promoter owns and operates the facility and collects revenues in order to repay the financing and investment costs, maintain and operate the facility and make a margin of profit. The concession period is usually between 15 to 30 years. In this partnership, the interests of both parties are met. The main interest of the public entity is to provide quality service delivery, for example a good road network, while that of the private entity is to earn profits on its investment.

It is important to note that the PPP option is not a recent approach, it had been proven over the years in many countries worldwide to be an effective tool in addressing financing gaps in infrastructural projects including roads. According to Cartlidge (2006), a PPP in the form of Build Operate and Transfer (BOT) was used for the construction of the Suez Canal as far back as 1858. A BOT project, basically, is one in which a public sector grants a concession to a private company for a fixed period of time. Other cases of PPP implementations exist in both the developed and developing economies.

A concession, in the pure sense, requires that total funding is provided by the private sector, with the repayment being guaranteed solely against the revenues of the project (Smith, 1999). It is, however, quite often reasonable, advantageous and strategic, under certain conditions, for governments to provide some necessary support in the form of subsidies or grants or guarantees as a way of minimizing project risks in order to attract prospective private investors.

Falam (2005) provided an account of the South African experience in PPP. Though the project was implemented successfully, it was not without some challenges. These challenges can be found in (Falam, 2005). Falam (2005) provides one of the few studies that identify PPP as an alternative source of road financing in Sub-Saharan Africa. Nonetheless, Falam (2005) by focusing on South Africa, a country which has a far stronger economy than the rest of Sub-Saharan Africa placed the study more in the bracket of "developed economies" than developing economies such as Ghana. Thus despite Falam's study informing our understanding of the relationship between PPP and alternative road financing, it remains that the study neglected the developing country perspective which is essential in understanding the interrelationship between PPP and developing economies.

Engel et al. (2003) traced the PPP experience of Chile in highways to 1993, and indicated that the country had effective tolling systems. Chile had commissioned over 3000 km of highways in Santiago deploying electronic tolling systems, and had over US$1.5 billion in project bonds being raised in international financial markets. The Urban Highway Concessions in Santiago comprised four projects. In this arrangement, the government guaranteed up to 70% of the investment plus costs of maintenance and operation of the projects, if the actual vehicular traffic demand fell significantly below the projected traffic.

Ghana is yet to experience the first successful PPP highway project. PricewaterhouseCoopers (2006) in their account enumerated a number of challenges to PPP in the highway sector in Ghana at the time. The Government of Ghana took a decision to extend private sector participation in the construction and maintenance of roads. This was expected to result in improved construction procedures and higher quality roads. There were many concerns raised by the main financiers of road projects, which comprised foreign investors, local investors and the Government of Ghana. The foreign investors were generally worried about the level of risk associated with political stability, the inadequacy of the existing legal and regulatory framework, and high profile disputes with international investors. For example, there were two separate disputes between Ghana and Ghana Telecom and Volta Aluminium Company. The local investors had concerns about the limited financial capacity of local financial institutions, lack of local technical expertise as well as the risk of late payments from government which imposes high working capital requirements. Notwithstanding these concerns, this study believes the highway construction will be viable for a concessionaire provided the total wealth invested in constructing the highway is less than total wealth generated from tolls within a given time horizon.

3. Methodology

In our research, we assume that four categories of vehicles will ply the road: light, medium, heavy and "others" vehicles and each pay an appropriate toll rate. Table 1 gives the type of vehicles that ply the Ghanaian highway and its description).

Table 1. Vehicle classes

Item	Vehicle type	Vehicle classes	Description
1	Cars/Taxis	Light	This category includes taxi, private or hired and saloon or estate cars.
2	Vans, Pick-Ups		This category includes pick-up, van, jeep, all cross country vehicles.
3	Small buses		Small buses include 19-32 seater buses, etc
4	Mammy Wagons/Medium buses	Medium	Mammy wagons are special trucks having wooden bodies for conveying both passengers and medium buses include 33-53 seater buses etc
5	Large Buses		Large buses include 54-seater buses and upwards
6	Light Trucks		This category comprises 2-axle truck with single rear wheel or 2-axle truck with twin rear wheels than 10 tons.
7	Medium Truck		This category comprises 2-axle wheels trucks with twin rear more than 10 tons.
8	Heavy Truck	Heavy	Heavy truck comprises 3-axle rigid truck, including tankers.
9	Semi-Trailers (Light)		These are semi-trailers with any configuration of 3-axles
10	Semi-Trailers (Heavy)		These are semi-trailers with any configuration of 4-axles.
11	Truck Trailers		These are large truck with any configuration of 5-axles.
12	Others	Others	Agricultural tractor and Agricultural tractor with trailer.

Three types of cost can be identified with highway operations: variable cost (minor maintenance cost, allowances, operating expenses such as stationery printing, electricity, diesel for standby generator, equipment repairs, etc.) that vary directly with the number of vehicles that use the road and fixed cost (wages of employees that man the tolls, the cost of manning the E card centre) that do not vary directly with the number of vehicles that ply the road. There is also a third cost, asset replacement cost that results from the cost of constructing the highway including the construction of bridges, the purchase of toll machines, the cost of engineering studies (i.e. design, feasibility studies and procurement). It is assumed that the asset replacement cost is paid off in a concessionary period of n months.

3.1. Derivation of the number of each of the different Sized Vehicle to Ply the Highway for a Specified Toll rates

t_f : The total cost (in dollars) of employee wages and maintenance of E-card Centre.

t_v : variable cost in dollars (minor maintenance cost; allowances; operating expenses such as stationary, printing, electricity, diesel for standby generator, equipment repairs, etc) that varies directly with the number of vehicles that use the road at every crossing.

s_l : Toll rate paid by light vehicles for using the highway.

t_{rl_i} : Total revenue from light vehicular toll payments.

n_{l_i} : The number of light vehicles that cross the toll payments booth in month l_i, i = 1, 2, 3, ..., n.

AR_c : Asset replacement cost (in dollars). This is the cost of constructing the highway, including the construction of bridges, the purchase of toll machines, and the cost of engineering studies.

w_i : The fraction of AR_c that has to be paid off in the ith month, $i \le w_i \le 1$ and $\sum_{i=1}^{n} w_i \equiv 1$.

We first assume only light vehicles ply the highway. Using the above definitions, the total revenues from light vehicle toll payments is:

$$t_{rl_i} \equiv n_{l_i} s_l$$

The monthly cost of operations can be approximated by

$(t_v - t_f) - w_i AR_c (1-r)^i$ where $t_v \equiv k_l n_{l_i}$.

t_v varies directly as the number of vehicles that ply the road, k_l is a constant that has to be determined. (Compare Andoh (2008), pg.78 where we have taken t_v as $n_p v_c$) . Hence, the profit for the ith month, P_{F_i}, on the light vehicular usage for month i is given by

$$P_{F_i} = n_{l_i} s_l - (k n_{l_i} + t_f) - w_i A R_c (1+r)^i$$

$w_i A R_c (1+r)^i$ is the strain on i^{th} month profit. Hence for profit to be realised for month i, n_{l_i} must be such that

$$n_{l_i} \geq \frac{t_f + w_i A R_c (1+r)^i}{s_l - k_l} \tag{1}$$

as we place no restriction on the number of vehicles that use the highway in any month.

In any month, we have a mix of light, medium, heavy and "others" vehicular usage and so the month profit, P_{F_i} for highway operations for month *i* will be given by

$$P_{F_l} = n_{l_i} s_l + n_{m_i} s_m + n_{h_i} s_h + n_{o_i} s_o - (k_l n_{l_i} + k_m n_{m_i} + k_h n_{h_i} + k_o n_{o_i} + t_f) - w_i A R_c (1+r)^i$$

where n_{m_i}, n_{h_i} and n_{o_i} are respectively the numbers of medium, heavy and "others" vehicles that use the highway for month i. s_m, s_h, and s_o are respectively the toll rates for medium, heavy and "others" vehicles. k_l, k_m, k_h, and k_o are constants that have to be determined for light, medium, heavy and "other" vehicles" respectively.

In practice, the total variable cost of operations, $k_l n_{l_i} + k_m n_{m_i} + k_h n_{h_i} + k_o n_{o_i} = T_v$, are not recorded separately for each vehicular type but as a lump sum. Of course, it would be costly to monitor every vehicle's contribution to this cost. Consequently, each vehicular type contribution will be obtained via

$$(w_l + w_m + w_h + w_o) T_v$$

where $w_l + w_m + w_h + w_o = 1$ and $w_l \geq 0$, $w_m \geq 0$, $w_h \geq 0$, $w_o \geq 0$. w_l, w_m, w_h, and w_o are weights for light, medium, heavy and "other" vehicles" respectively. Hence, the constants can be determined by

$$k_l n_{l_i} = w_l T_v, k_m n_{m_i} = w_m T_v, k_h n_{h_i} = w_h T_v, k_o n_{o_i} = w_o T_v \tag{2}$$

We use the average weight of each vehicle class and the frequency of toll booth crossing as proxy for weights assignments. The reason is that the heavier the vehicle, the more likely it will cause damage to the highway and the greater the frequency of usage of the highway the more likely a vehicle will cause damage to it.

Incorporating (2) into the expression for P_{F_i} above, the expression reduces to

$$P_{F_l} = n_{l_i} s_l + n_{m_i} s_m + n_{h_i} s_h + n_{o_i} s_o - T_v - t_f - w_i A R_c (1+r)^i$$

Hence, for the PPP arrangement to be viable for month i, $P_{F_i} > 0$. For fixed toll rates s_l, s_m, s_h, and s_o, the objective of any concessionaire is to maximize P_{F_i} for month i, i = 1, 2, ...,12. This can summarized as

Maximize $P_{F_I} = n_{l_i} s_l + n_{m_i} s_m + n_{h_i} s_h + n_{o_i} s_o - T_v - t_f - w_i AR_c (1+r)^i$

Subject to:

$$n_{l_i} s_l + n_{m_i} s_m + n_{h_i} s_h + n_{o_i} s_o > T_v + t_f + w_i AR_c (1+r)^i$$
$$0 \le n_{l_i} \le \alpha_i$$
$$0 \le n_{m_i} \le \beta_i \qquad (3)$$
$$0 \le n_{h_i} \le \gamma_i$$
$$0 \le n_{o_i} \le \delta_i$$

α_i, β_i, γ_i and δ_i are respectively the maximum number of light, medium, heavy and "other" vehicles that use the highway for month i. This problem can be formatted into a spreadsheet and the spreadsheet software can be instructed to compute the solution (see Brandimarte (2002), chapter 3, Bertsimas & Freund (2004), chapter 7 or Winston (2004), pg 204).

3.2. Toll rate for a given number of light, medium, heavy and "others" vehicles that ply the highway

If we assume that only light vehicles use the highway and hold the number of light vehicular usage fixed for month i, then the toll rate, s_l, for light vehicular movement for the i^{th} month has to exceed

$$s_l \ge k_l + \frac{t_f + w_i AR_c (1+r)^i}{n_{l_i}} \qquad (4)$$

If we hold the number of vehicular usage fixed, then the profit for month i is

$$P_{F_i} = n_{l_i} s_l + n_{m_i} s_m + n_{h_i} s_h + n_{o_i} s_o - T_v - t_f - w_i AR_c (1+r)^i$$

Hence for viability, the toll rates has to be set such that $P_{F_i} > 0$. The optimal toll rates s_l, s_m, s_h, and s_o for all four types of vehicles that use the highway for month i can be determined from the following linear programming problem:

Maximize $P_{F_I} = n_{l_i} s_l + n_{m_i} s_m + n_{h_i} s_h + n_{o_i} s_o - T_v - t_f - w_i AR_c (1+r)^i$

Subject to:

$$n_{l_i}s_l + n_{m_i}s_m + n_{h_i}s_h + n_{o_i}s_o > T_v + t_f + w_i AR_c(1+r)^i$$

$$0 \le s_l \le \tilde{\alpha}_i$$

$$0 \le s_m \le \tilde{\beta}_i \tag{5}$$

$$0 \le s_h \le \tilde{\gamma}$$

$$0 \le s_o \le \tilde{\delta}_i$$

$\tilde{\alpha}_i$, $\tilde{\beta}_i$ $\tilde{\gamma}_i$ and $\tilde{\delta}_i$ are the maximum permissible charges for month i for light, medium, heavy and "others" vehicles respectively. Again, the objective of any rational concessionaire is to maximize P_{F_i} for each month i, i = 1, 2, ...,12.

4. Empirical results

4.1. Assumptions and Data Analysis

Data for the empirical study, analysis, and illustration of our model were obtained from the Ghana Highway Authority. This includes data on the cost of various road construction types and traffic data for trunk roads obtained from 29 toll booths scattered across Ghana for the year 2010. Ghana Highway Authority is charged with the responsibility for the administration, development and maintenance of trunk roads and related facilities in Ghana. This includes the management of all toll booths scattered throughout Ghana.

We assume that the toll booth will be constructed 50km apart and that higher portions of AR_c will be paid in earlier years than later years. The reason for this latter assumption is the fact that government has come to the realization that mass transportation is the way for the future and plans are underway to develop the rail sector. If the sector becomes fully operational it is envisaged that large portions of the population are likely to go by rail. It is also better to pay off higher portions of AR_c in earlier years as a delay will mean higher toll rates in latter years by virtue of the interest rate component. The 50km assumption is a government policy for tolling for trunk roads construction but can be subject to sensitivity analysis. We also assume that the highway will be constructed single carriageway and the type of construction is reconstruction. Analysis for the other road types such as dual carriageway and motorway can be accomplished in a similar fashion. The type of construction and the corresponding cost per kilometer can be found in Table 2. Also the cost of toll structure and accompanying automation cost for each road type can be found in Table 3. A summary of the variable and fixed costs of operations are summarized in Table 4.

Table 2. Type of construction and associated cost

Construction type	Cost per km (US dollars)
Reconstruction (Asphaltic concrete)	1500000
Upgrading	839150
Rehabilitation	350000

Table 3. Cost of road toll booth structure and automation

Cost of road toll booth structure and automation			
	Cost of toll	Automation	Total cost
Road type	structure (US$)	cost (US$)	(US$)
Single carriageway	747839.71	29163.97	777003.67
Dual carriageway	1309497.74	58327.93	1367825.68
Motorway	1309497.74	235255.99	1544753.74

Table 4. Cost categories for PPP arrangement

Cost type	Components	Amount (GH¢)	Amount (US$)
Variable	Operating expenses	29330	19008.42
	Total	29330	19008.42
Fixed	Staff salaries and allowances	48242.50	31265.39
	Manning of E-card centre	1400	907.32
	Management fee	10500	6804.93
	Total	60142.50	38977.64
AR_c	Single carriageway		
	(Structure + automation)		777003.67
	50km asphaltic concrete		$\underbrace{50 \times 1500000}_{75000000}$
	50km upgrading		$\underbrace{50 \times 839150}_{41957500}$
	50km rehabilitation		$\underbrace{50 \times 350000}_{17500000}$
	50km regravelling		$\underbrace{50 \times 135000}_{6750000}$

Figure 1 shows the daily average number of vehicle movements on toll bridges and roads across Ghana collected in November 2010. Clearly, the number of light vehicular usage outnumbers the usage of medium, heavy and "other" vehicles. It is also clear from the plot that to reduce the congestion on the roads in Ghana, mass transportation has to be encouraged. A number of benefits flow from this observation: less funds will be spend on highway maintenance, fewer vehicles can convey passengers and goods at a faster rate as the road users decline and less accidents as the number of vehicles that use the highway decline. Besides, vehicle operational cost and travel time will reduce.

Figure 2 shows boxplot of the average daily vehicular movement at toll booths in Ghana in 2010. It is clear that Accra and its environment lead in vehicular movement and therefore an appropriate area for PPP arrangement.

4.2. Data Analysis

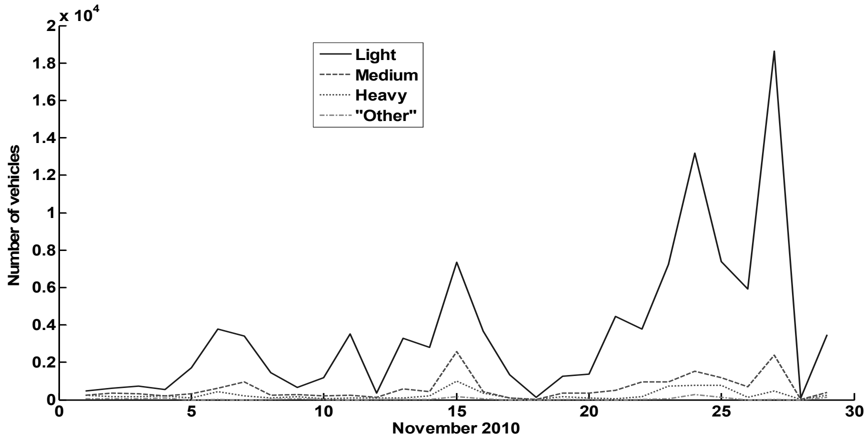

Figure 1. The average number of daily traffic reported in November 2010.

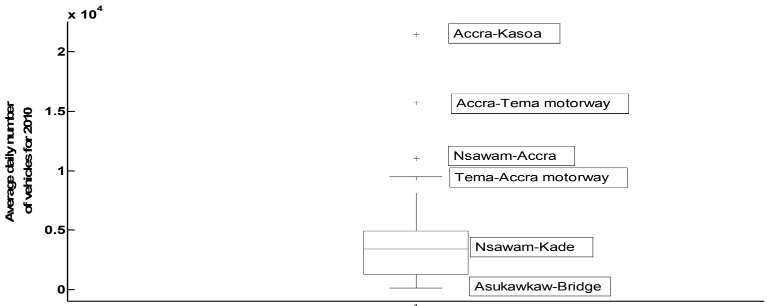

Figure 2. Boxplot of average daily vehicular movement at toll booths in Ghana for 2010.

If the highway is to be reconstructed and we assume light vehicular usage on the highway only, then from equation (1) and the first part of equation (2) in section 3.1, it can be seen that for profit to be realized in month i,

$$n_{l_i} > \frac{w_l T_v + t_f + w_i A R_c (1+r)^i}{s_l} \tag{6}$$

Table 5 shows how the weight for each vehicle class is assigned or calculated. From Table 5, we see that $w_l = 0.501$, $w_m = 0.151$, $w_h = 0.333$, and $w_o = 0.015$

Substituting values for the parameters on the right-hand side (RHS) of equation (6) from the table, we have that

$$n_{l_i} \geq \frac{1}{0.49}\left[0.501\,x\,19008.42 + 38977.64 + 75777003.67x\ 1.0041666^{i}\,w_i\right] \quad (7)$$

where we have chosen 5% interest rate per annum (compare PPIAF-World Bank (2006)). If higher portion of AR_c are paid in earlier years than latter years, then it can be demonstrated graphically that the number of vehicles that must use the highway for the concessionaire to realize profit declines with increasing concessionary period. What this means is that the current toll rate is inadequate for a number of years for all concessionary periods.

Table 5. Total weight on highway in November 2010

Type of vehicle	Average weight of vehicle (kg)	Median number of vehicular crossing on toll booth	Total weight on highway (kg)	Weight assigned to each vehicular class
Light	2057.97	2798	5758200.05	0.501
Medium	4717.35	367	1731267.45	0.151
Heavy	22185.379	173	3838070.57	0.333
"Others"	12891	13	167583.0	0.015
Totals			11495121.08	1

On the other hand, the toll rate, s_1, for light vehicular movement for month i is

$$s_l > \frac{1}{n_{l_i}}\left[0{,}501\,\mathrm{x}\,19008.42 + 38977.64 + 75777003.67\,\mathrm{x}\,1.0041666^{\mathrm{i}}\,w_i\right] \quad (8)$$

In 2010, the daily average number of light vehicles that used the highways around the various regions lies in [91,18616], the highest occurring in Accra-Kosoa toll booth and the lowest occurring in the Asukawkaw toll booth in the Volta region of Ghana (see the boxplot in figure 2). Assuming average daily number of light vehicular usage lies in [2798,18616] within the concessionary period, then the appropriate price for viability for month i lies in

$$\left[0.05 + 135.68\,\mathrm{x}\,1.0041666^{\mathrm{i}}\,w_i,\ 0.35 + 902.75\,\mathrm{x}\,1.0041666^{\mathrm{i}}\,w_i\right]$$

Of course, no highway would be constructed for which only light vehicles will use. Therefore, with all types of vehicles using the highway, the objective of any concessionaire is to maximize the profit P_{F_i} for month i, i = 1, 2, ..., 12 as follows:

Maximize $P_{F_i} = 0.49n_{l_i} + 0.97n_{m_i} + 1.54n_{h_i} + 0.32n_{o_i}$ - $57986.06 - 75777003.67x\,1.0041666^{\mathrm{i}}\,w_i$

Subject to:

$0.49n_{l_i} + 0.97n_{m_i} + 1.54n_{h_i} + 0.32n_{0_i} > 57986.06 + 75777003.67\,\mathrm{x}\,1.0041666^{\mathrm{i}}\,w_i$

$0 \leq n_{l_i} \leq 558480$ (9)

$0 \le n_{m_i} \le 77490$

$0 \le n_{h_i} \le 29670$

$0 \le n_{o_i} \le 8160$

The upper bounds in the last four constraints are the highest number of vehicles of each vehicle class that used the toll booth in Ghana recorded in November 2010. We used Matlab to solve the problem. Figure 3 shows the optimal profit for the concessionary periods 15, 30, 45 and 60 years. It is clear from the figure that the potential loss of the concessionaire decreases with increasing concessionary period and there is less variability in the optimal profit for greater concessionary periods. For fair toll rates, this should translate to lower toll rates for road users for greater concessionary periods.

For a 15-year concession for example, in the first month of operation, an optimal profit of -\$335665.33 occurs at (n_{l_i} = 734056, n_{m_i} = 253066, n_{h_i} = 205246, n_{o_i} = 183736). In the second month, an optimal profit of -\$332466.65 occurs at ($n_{l_i} = 732382$, n_{m_i} = 251392, n_{h_i} = 203572, $n_{o_i} = 182062$). As the optimal profit increases as can be seen from the plot, the number of each sized vehicles declines until the 105th month. From the 106th month and beyond, though the optimal profit increases, it (the optimal profit) is always obtained at ($n_{l_i} = 558480$, n_{m_i} = 77490, n_{h_i} = 29670, n_{o_i} = 8160) which are the upper bounds of the decision variables of the optimization problem.

For a 30-year concession, in the first month of operation an optimal profit of -\$106825.05 occurs at (n_{l_i} = 614357, n_{m_i} = 13336, n_{h_i} = 85547, n_{o_i} = 64037). As the optimal profit increases, the number of each sized vehicle that must use the highway declines until the 178th month. From the 179th month onwards, though the optimal profit increases, the optimal point always occurs at (n_{l_i} = 558480, n_{m_i} = 77490, n_{h_i} = 29670, n_{o_i} = 8160), which are the upper bounds of the decision variables.

Now for a 45-year concession, in the first month of operation an optimal profit of -\$30120.91 is obtained at (n_{l_i} = 574236, n_{m_i} = 93246, n_{h_i} = 45426, $n_{o_i} = 23916$). As the optimal profit increases as depicted on the figure, the number of each sized vehicle that must use the highway declines until the 255th month. From the 256th month onwards, though the optimal profit increases, it is always obtained at (n_{l_i} = 558480, n_{m_i} = 77490, n_{h_i} = 29670, $n_{o_i} = 8160$) which are again the upper bounds of the decision variables.

Finally, in the first month of operation, for a 60-year concession, an optimal profit of \$22,743.83 was obtained at (n_{l_i} = 558480, n_{m_i} = 77490, n_{h_i} = 29670, $n_{o_i} = 8160$). There is then a decline in the optimal profit until the 55th month with the optimal point occurring at (n_{l_i} = 558480, n_{m_i} = 77490, n_{h_i} = 29670, $n_{o_i} = 8160$), in each case. In the 56th month of operation, an optimal profit of -\$42.16 occurs at (n_{l_i} = 558503, n_{m_i} = 77513, n_{h_i} = 29693, n_{o_i} = 8183). From this month onwards, the optimal profit further declines reaching its lowest value of -\$15460.17 at 240th month with optimal point occurring at (n_{l_i} = 566567, n_{m_i} = 85577, n_{h_i} = 37757, n_{o_i} = 16247). This can be seen in the lower right plot of figure 3 in which the optimal profit declines in earlier years and begin to rise again around month 400. From the 387th months onwards, though the optimal profit increases, the optimal point always occur at the end point of the optimization problem i.e. ($n_{l_i} = 558480$, $n_{m_i} = 77490$, $n_{h_i} = 29670$, $n_{o_i} = 8160$).

In general, the greater the years of concession, the smaller the number of vehicles that must use the highway for optimal profit.

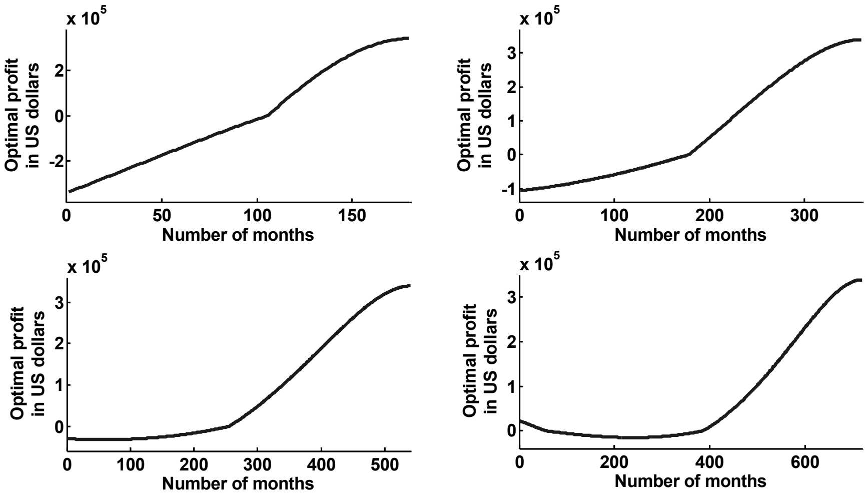

Figure 3. Optimal profit with increasing concessionary period.

The graph at the upper left of figure 3 represents 15 year concessionary period, the one at the upper right represents 30 year concessionary period. That at the lower left represents 45 year concessionary period, while the one at the lower right represents 60 year concessionary period.

A concessionaire often likes to determine the prices that will enable him or her to make profit. The following model enables a concessionaire to determine the optimal toll rates that maximize his/her profit.

Maximize $P_{F_i} = 558480 s_l + 77490 s_m + 29670 s_h + 8160 s_o - 57986.06 - 75777003.67 \text{ x } 1.0041666^i w_i$

Subject to:

$$558480 s_l + 77490 s_m + 1.0041666^i w_i > 57986.06 + 75777003.67 \text{ x } 1.0041666^i w_i$$

$$0 \le s_l \le 0.49$$

$$0 \le s_m \le 0.97 \qquad (10)$$

$$0 \le s_h \le 1.54$$

$$0 \le s_o \le 0.32$$

The upper bounds in the last four constraints are the maximum toll charges for each vehicular type. It should be noted that we have permitted all highways for which the average daily vehicular usage exceeds the median number of daily average vehicular usage for 2010 to be potential PPP candidate. Consequently, we obtained model (10) using $n_{l_i} = 83940$, $n_{m_i} = 11010$, $n_{h_i} = 5190$, $n_{o_i} = 390$. The model was solved using Matlab

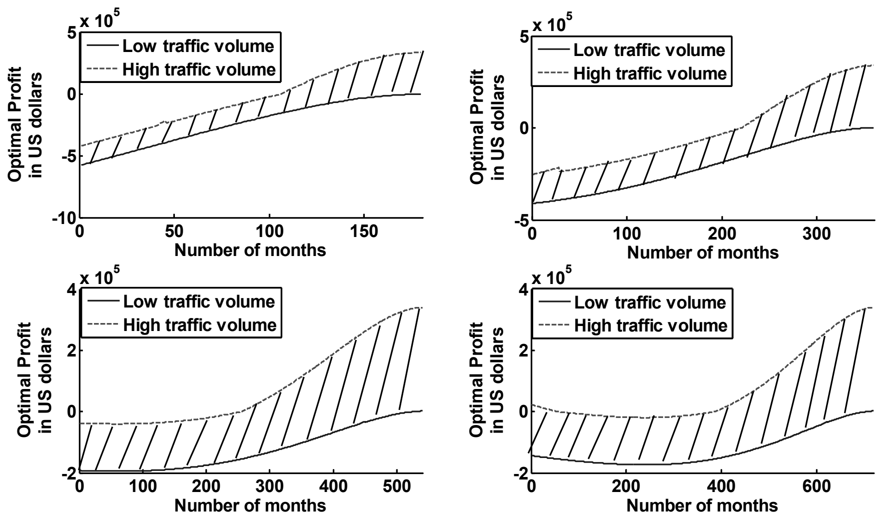

Figure 4. Optimal profit with increasing concessionary periods for low traffic volume (solid line) and high traffic volume (dashed line).

The graph at the upper left of figure 4 represents 15 year concessionary period, the one at the upper right represents 30 year concessionary period. That at the lower left represents 45 year concessionary period, while the one at the lower right represents 60 year concessionary period.

It is clear from figure 4 that the optimal profit improves with increasing concessionary period and the potential loss to a concessionaire based on the current toll rates is greater with shorter concession periods. In addition, the optimal profit improves with increasing traffic volume. For all traffic volume within the interval:

$n_{l_i} \in [83940, 558480]$, $n_{m_i} \in [11010, 77490]$, $n_{h_i} \in [5190, 29670]$ $n_{o_i} \in [390, 8160]$

the optimal profit will fall within the shaded part of figure 4.

In the first month of a 15-year concessionary period, when traffic volume is low (solid line), the optimal profit of -\$574,662.16 (has value -\$418,816.07 when traffic volume are high) was obtained with optimal values \$7.27, \$7.75, \$8.32, \$7.10 for the toll rates s_l, s_m, s_h, and s_o respectively. But when traffics are high, the optimal profit of -\$418,816.07 was obtained with the decision variable (toll rates) values \$1.24, \$1.71, \$2.28 and \$1.06 respectively.

As the optimal profit increases, the optimal toll rate at which it is obtained continued to decrease until the 176th when the optimal profit of -\$30.84 was obtained at the optimal toll rates \$0.49, \$0.97, \$1.54 and \$0.32. From the 177th month onwards, the optimal profit continued to be positive and was always obtained with the optimal toll rates \$0.49, \$0.97, \$1.54 and \$0.32, which are the upper bounds of the decision variables s_l, s_m, s_h, and s_o respectively.

When the traffic volumes are high and the optimal profit increases, the optimal toll rates with which they are obtained continues to decreases monthly until the 105th month when an optimal profit of -$2906.61 was obtained with optimal toll rates $0.50, $0.98, $1.55 and $0.33 for s_l, s_m, s_h, and s_o respectively. From the 106th month onwards, the optimal profit continued to be obtained at the optimal toll rates $0.49, $0.97, $1.54 and $0.32, which are the upper bounds of the decision variables s_l, s_m, s_h, and s_o respectively.

Similar observations can be made with the 30 and 45 years concession periods.

In the first month of operation of a 60 –year concession, when traffic volume is low (solid line) an optimal profit of -$143898.65 ($22,743.83 when traffic volume is high) occurs at ($2.19, $2.67, $3.24, $2.02). The optimal profit then continued to decline with a corresponding increase in the optimal toll rates until the 267th month in which an optimal profit of -$173,096.62 (-$18,721.19 when traffic volume is high) was obtained with optimal toll rates $2.53, $3.01, $3.58 and $2.36.

From the 268 month onwards, the optimal profit improves with a corresponding decrease in the optimal toll rates reaching its highest value of $1929.51 ($339,125.31 when traffic volume is high) in the 720th month with optimal toll rates $0.49, $0.97, $1.54 and $0.32, which are the upper bounds of the decision variables s_l, s_m, s_h, and s_o respectively. When the traffic volume is high, the optimal profit is always obtained at $0.49, $0.97, $1.54 and $0.32 until the 55th month. From the 56th month, there is a gradual increase in the optimal toll rates until 264th month with the optimal toll rates $0.52, $1.00, $1.57 and $0.35.

Thereafter, there is a continuous decline in the optimal toll rate until the 386th month when the optimal toll rates values $0.49,$0.97,$1.54 and $0.32 were obtained. From the 386th month onwards, the optimal profit was always obtained with the optimal toll rates $0.49, $0.97,$1.54, and $0.32, which are upper bounds of the decision variables s_l, s_m, s_h, and s_o respectively.

In general, toll rates has to be higher in earlier years of the concession period and the greater the number of years of concession, the lower the toll rate.

4.3. Sensitivity Analysis

Now, we investigate the effect of changes in the number of each vehicular class that uses the highway on the optimal profit. Let $\alpha_i \in [2730, 658480]$, $\beta_i \in [390, 87490]$, $\gamma_i \in [330, 39670]$, $\delta_i \in [0, 9160]$ We use the interval steps 21858.37, 2903.37, 1311.37 and 305.36 to generate different values for α_i, β_i, γ_i and δ_i within their respective closed intervals defined above. For example, the values generated for α_i within the closed interval [2730, 658480] are obtained as follows:

2730, 2730 + 21858.37 = 24588.37, 24588.37 + 21858.37 = 46446.74, 46446.74 + 21858.37 = 68305.11, ..., 6584800 (658480 being the upper bound for the values generated).

The various sets of values generated for α_i, β_i, γ_i and δ_i respectively within their respective closed interval were each used as the upper bounds in the bound constraints for n_{l_i}, n_{m_i}, n_{h_i}, and n_{o_i} respectively in model (9) and the resulting optimization problem obtained in each case was solved to obtain the optimal profit.

As can be seen from figure 5 (and for each concessionary period), the optimal profit improves with increasing number of each sized vehicle that uses the highway. In addition, it can be seen from figure 5 that the potential loss to the concessionaire declines with increasing concessionary period, making the optimal profit more predictable for greater concessionary periods.

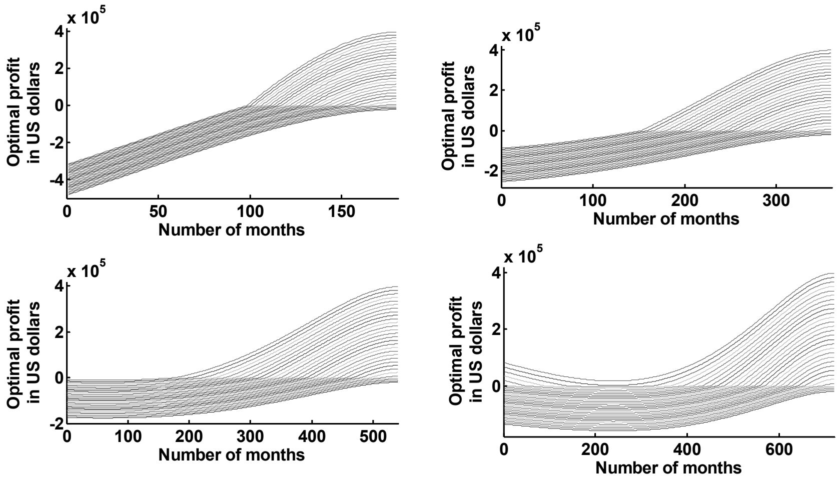

Figure 5. Optimal profit with increasing concessionary periods and increasing values of α_i, β_i, γ_i and δ_i.

The graph at the upper left of figure 5 is based on 15-year concessionary period with increasing upper bound values α_i, β_i, γ_i and δ_i for the decision variables n_{l_i}, n_{m_i}, n_{h_i}, and n_{o_i} respectively. The one at the upper right is based on 30-year concessionary period with increasing α_i, β_i, γ_i and δ_i for the decision variables n_{l_i}, n_{m_i}, n_{h_i}, and n_{o_i} respectively. That at the lower left represents 45-year concessionary period, also with increasing upper bound values α_i, β_i, γ_i and δ_i for the decision variables. The graph at the lower right is based on a 60-year concessionary period with increasing upper-bounds values for the decision variables.

Next, we investigate the effect of changes in the toll rates for each sized vehicle on the optimal profit. To do this, we let

$\tilde{\alpha}_i \in [0, \$14.5]$, $\tilde{\beta}_i \in [0, \$14.5]$, $\tilde{\gamma}_i \in [0, \$29]$ and $\tilde{\delta}_i \in [0, \$7.25]$

and using the interval steps 0.5167 for the first two intervals, 1 and 0.275 for the third and fourth intervals and changing the values of the upper bounds $\tilde{\alpha}_i$, $\tilde{\beta}_i$ $\tilde{\gamma}_i$ and $\tilde{\delta}_i$ (I = 1, 2, 3, ……,30) of the toll rates of the different sized vehicles and also changing the numbers n_{l_i}, n_{m_i}, n_{h_i}, and n_{o_i} of the different sized vehicles models (5) or (10), we obtain the different case problems from the optimization problem in (10) of section 4. The graphs in Figure 6 are based on the solutions to these problems.

The graph at the upper left of figure 6 is based 15-year concessionary period with increasing upper bound values $\tilde{\alpha}_i$, $\tilde{\beta}_i$ $\tilde{\gamma}_i$ and $\tilde{\delta}_i$ for the decision variables s_l, s_m, s_h, and s_o respectively. The one at the upper right is based on 30-year concessionary period with increasing $\tilde{\alpha}_i$, $\tilde{\beta}_i$ $\tilde{\gamma}_i$ and $\tilde{\delta}_i$ for the decision variables s_l, s_m, s_h, and s_o respectively. That at the lower left represents 45-year concessionary period, also with increasing $\tilde{\alpha}_i$, $\tilde{\beta}_i$ $\tilde{\gamma}_i$ and $\tilde{\delta}_i$ for the decision variables. The graph at the lower right: is based on 60- year concessionary period with increasing upper bounds for the decision variables.

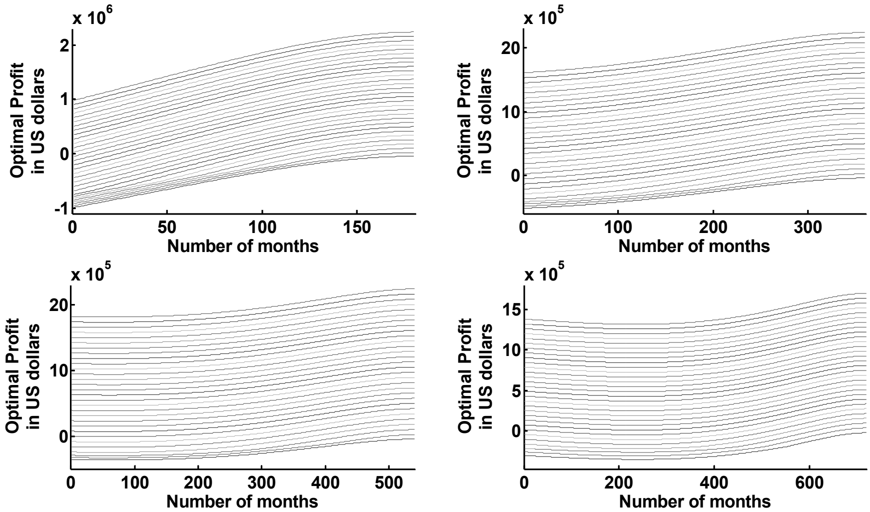

Figure 6. Optimal profit with increasing concessionary periods and increasing values of $\tilde{\alpha}_i$, $\tilde{\beta}_i$ $\tilde{\gamma}_i$ and $\tilde{\delta}_i$

As expected, as the toll rates for each sized vehicle increased, the optimal profit also increased for each concessionary period. As the concession period increases, the potential loss to the concessionaire declines. These results show that the fair thing is for road users to pay lower rates for greater concessionary period.

We next investigate how much should a concessionaire pay for the PPP to ensure optimal profit at all times? We assume that all the variables and the fixed costs are borne by concessionaire. Our task is to determine the portion of AR_c that should be borne by the concessionaire to realize some profit. To obtain an estimate of this proportion, $\alpha \in [0,1]$ write the P_{F_i} as

$$P_{F_i} = \tilde{n}\tilde{s} - T_v - t_f - \alpha(w_i AR_c (1+r)^i)$$

where $\tilde{n}$ is the number of all vehicles (i.e. light, medium, heavy and "others" together) and $\tilde{s}$ is the toll rate for each vehicle that passes through a toll gate. For viability, we must have that:

$$\tilde{n} \geq \frac{T_V + t_f + \alpha(w_i AR_c (1+r)^i)}{\tilde{s}}$$

We verified our estimate of α by solving the following optimization problem:

Maximize $P_{F_i} = 0.49n_{l_i} + 0.97n_{m_i} + 1.5n_{h_i} + 0.32n_{o_i}$ - 57986.06 –
α (75777003.67 x $1.0041666^{i}\, w_i$)

Subject to:

$$\begin{aligned} &0.49n_{l_i} + 0.97n_{m_i} + 1.5n_{h_i} + 0.32n_{o_i} > 57986.06 + \alpha(75777003.67 \times 1.0041666^{i} w_i) \\ &0 \leq n_{l_i} \leq 558480 \qquad (11) \\ &0 \leq n_{m_i} \leq 77490 \\ &0 \leq n_{h_i} \leq 29670 \\ &0 \leq n_{o_i} \leq 8160 \end{aligned}$$

For all appropriately chosen α, the optimal profit will lie in the first quadrant of the coordinate axes. Figure 7 shows the estimate of the number of vehicles that can be hauled for the concessionary period 15, 30, 45 and 60 years. The results show that the percentage of AR_c that can be borne by a concessionaire increases with the number of years of the concession. With a 15 year concession, a PPP arrangement is viable on highways for which the number of vehicular usage lie in the upper half of the solid line (figure 7, upper left). A concessionaire has to contribute about 3.7% of AR_c. With 30, 45 and 60 years concessionary periods, a concessionaire has to contribute about 7.4%, 11% and 12.3% of AR_c for sustainability.

The graph at the upper left of figure 7 represents 15-year concessionary period, the one at the upper right represents 30-year concessionary period. That at the lower left represents 45-year concessionary period, while the one at the lower right represents 60- year concessionary period. In this case, the appropriate toll rates for the 15, 30, 45 and 60 year concessionary periods can be deduced from solving the optimization problem in section 3.2. It should be noted that the percentage of AR_c that the concessionaire can accommodate in order to realise some profit increases on highways with increasing vehicular usage. For example, for the Accra-Kasoa toll booth, a concessionaire can contribute about 27.9%, 55.6%, 82.1% and 92% of AR_c for the concession periods 15, 30, 45 and 60 years respectively in addition to the variable and fixed cost to realize some profit.

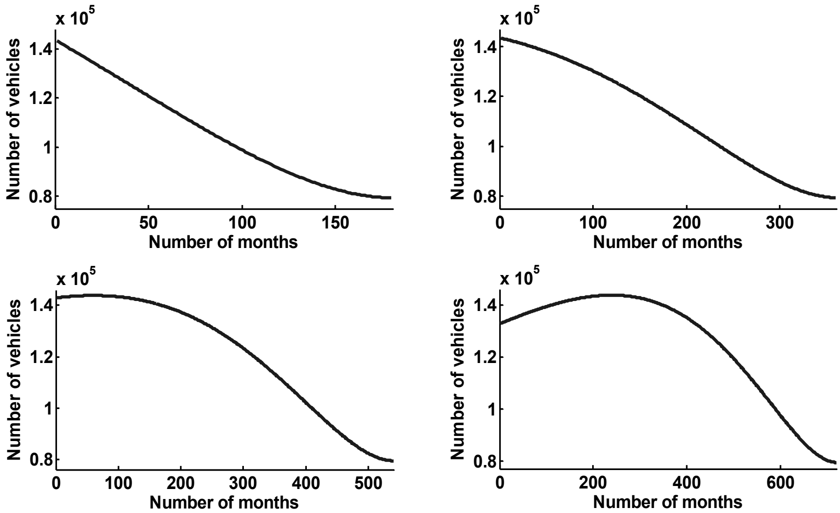

Figure 7: Estimate of the number of vehicles that can be hauled for different concessionary periods.

We performed similar analysis for the case where the highway is upgraded and rehabilitated and obtained the results shown in Table 6.

Table 6. Type of road and the percentage contribution of ARc by a concessionaire for varying years of concession.

Type of construction	Amount of AR_c a concessionaire may contribute to realise profit			
	Years of concession			
	15 years	30 years	45 years	60 years
Upgrading	6.6%	13.2%	19.5%	21.9%
Rehabilitation	15.5%	30.5%	45.5%	51%

5. Conclusions

The mathematical model developed in this paper is a very good and useful planning and decision-making tool for any business man or organization that wants to enter into partnership with any government in highway construction and management. The model determines the optimal profit, the optimal number of different vehicle sizes, and the optimal toll rates for a given highway concessionary period.

Based on the analyses done with the model, using the data collected from the Ghana Highway Authority, we conclude that PPP arrangement is possible on a number of highways in Ghana. The greater the number of years of concession, the less the construction costs borne by the government. There is greater variability in the optimal profit with shorter concessionary periods than with longer concessionary periods. In addition, the greater the number of years of concession, the lower should be the toll rate for the road user. PPP arrangement in the pure sense can be viable for a concessionaire for a few selected road provided the number of years of concession are long.

We recommend that a centre for PPP expertise should be created to provide guidance to all public authorities on the procurement and drafting of PPP projects and contracts. A review of the structure and management of the trunk road network in Ghana should be performed, with the view of finding areas where PPP can best fit in, and thereby allowing its potential benefits to be harnessed to the advantage of the nation. In addition, monthly income from AR_c has to be put in a fund, managed by a fund manager. Asset that breaks down has to be funded from this fund including major maintenance costs.

Additional research has to be conducted on dual carriageways and motorways to determine how much a concessionaire must contribute for viability. Besides, further investigation has to be conducted to find out the willingness of local financial institutions to participate in a PPP project financing.

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